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Abstract

This paper characterises efficiency-enhancing taxes when job search activities generate excessive mismatch between firms and workers. Workers can direct and modulate their search efforts towards particular firms before matching and bargaining a wage. A composition externality arises because workers do not internalise the consequences of mismatch on the entry decision of firms, resulting in lower average sectoral productivities and suboptimal job creation. The optimal tax scheme combines a wage tax to restore an efficient surplus sharing, and a job surplus tax to make job seekers more selective. Taxation is anti-redistributive for several ways of modelling heterogeneity.

JEL Code: H21, H23, J24, J42

Keywords: anti-redistributive taxation, composition externality, job quality, mismatch, search strategy

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Introduction

This paper characterises efficiency-enhancing taxes when job search activities generate excessive mismatch between firms and workers.

The frictional nature of the labour market has generated new results in public finance research. How search frictions shape wages and unemployment matters for designing public policies such as redistribution taxes (Hungerbühler et al., 2006; Lehmann et al., 2011; Golosov et al., 2013; Bagger et al., 2018), or unemployment insurance (Cremer et al., 1995; Geromichalos, 2015; Boadway and Cuff, 2018; Landais et al., 2018). Most of this research, however, abstracts from the inherent inefficiency of frictional markets through specific assumptions. In the presence of search frictions, laissez-faire equilibria are generally inefficient because agents generate positive and negative externalities from searching and matching. To preserve efficient equilibria, the aforementioned literature relies on specific assumptions such as homogenous agents or perfect market segmentation. In these cases, there is no Pigouvian motive for public intervention, and optimal policies only serve an equity motive.

The present paper investigates the Pigouvian motive of optimal taxation in a labour market with two-sided heterogeneity, partial market segmentation and unobserved search efforts. At the laissez-faire equilibrium, firms and workers are inefficiently mismatched because job seekers are not selective enough. Absent of equity motive, the efficiency-enhancing tax scheme has anti-redistributive characteristics. Ignoring this inefficiency can therefore understate the deadweight loss of redistributive policies.

The model builds on the framework of Diamond-Mortensen-Pissarides (see Pissarides, 2000, for an overview) with firm and worker heterogeneity. The search-and-matching technology is modelled with two main assumptions. First, the labour market is partitioned by firm type. Each firm type defines a segment of the labour market in which matching is random. Workers' search decisions are informed by firm types. Wages are Nash-bargained after matching. I thus generalise the two-sector model of Uren (2006) and the horizontal heterogeneity model of Decreuse (2008) to an environment with unrestricted two-sided heterogeneity. Second, workers can search simultaneously on several segments, and they can modulate the intensity of search accordingly. The search technology has decreasing marginal returns within each segment of the labour market, as in the directed search model of Decreuse and Zylberberg (2011).

The economic mechanism in this article relies on a general equilibrium effect which connects workers' search strategies to firms' decisions to create jobs. A job seeker has incentives to search for jobs that are not their best fit in order to leave unemployment quickly. Job seekers' selectivity thus determines the extent of mismatch on the labour market. From the firms' perspective, job mismatching deteriorates profits because jobs are on average less productive. Consequently, more mismatch leads to fewer job openings. By not internalising the firms' job creation costs, workers are not selective enough and thus job quality is diminished. This *composition externality* adds to the standard search externalities of the Diamond-Mortensen-Pissarides framework. The laissez-faire equilibrium is always inefficient. This inefficiency, already described by Uren (2006) and Decreuse (2008), is considered in a richer environment.

The contribution of this research is to characterise the system of taxes that restores the efficiency of the decentralized equilibrium in a general framework with rich heterogeneity and search technology. Optimal taxes describe the departure from the efficient allocation. In this article, Proposition 1 proves the existence of pay-off profiles that decentralise efficient job creation and efficient search efforts. Proposition 2 shows that decentralised Nash-bargained pay-offs without taxes are not one of them, implying that the laissez-faire equilibrium is inefficient in general. Proposition 3 defines the optimal taxation scheme that decentralises the efficient allocation. The optimal tax scheme consists in redistribution from low-surplus to high-surplus jobs within each segment of the labour market. Workers are thus provided with the incentives to be more selective.

Optimal taxes have two salient features. First, the government can decentralise the efficient first-best allocation *without* observing the multi-dimensional search strategies. A proportional tax on wages, a proportional tax on match surplus and a lump-sum tax corrects for two inefficiencies of the labour market. The proportional tax on wages maintains the Hosios-Pissarides condition (Hosios, 1990; Pissarides, 2000), which states that the match surplus is split according to the search externalities created by each side of the market (Boone and Bovenberg, 2002). The proportional tax on match surplus makes good matches more profitable to workers. As a consequence, job seekers are more selective in their applications as they internalise the composition externality.

Second, the optimal tax scheme depends on a limited number of variables, namely the bargaining power, the matching elasticity, and the match surpluses. The optimal tax scheme also extends to the presence of on-the-job search. A match with a previously unemployed worker generates more surplus than one with a previously employed one. Therefore, the government puts more fiscal pressure on employed workers to reduce their search efforts. For specific modellings of heterogeneity, redistribution from low-surplus to high-surplus job is equivalent to redistribution from low-productivity to high-productivity job, what I define as anti-redistribution.

The model is calibrated to discuss occupational mismatch in the United States. The net output loss of the laissez-faire equilibrium compared to the social optimum ranges from 2% to 15% depending on the matching elasticity and the bargaining power of workers. The correlation between the level of optimal taxes and job productivity is below -0.5, suggesting that the efficiency-enhancing tax scheme is anti-redistributive. Occupational mismatch in this context differs from the concept introduced by Sahin et al. (2014). Mismatch comes from the productivity loss from inadequate matches, whereas these authors consider the inadequacy between the number of job seekers and the number of vacancies by occupation.

This paper is related to the public finance literature that features search frictions. The article by Bagger et al. (2018) is the closest paper in this literature. In their framework, heterogeneous workers search off and on-the-job to climb a job ladder. By taxing productive workers, the government distorts search efforts and generates inefficiency. There is no job vacancy creation in their model, and so no composition externality. The equity motive is the only reason for introducing taxes. I complement their approach by accounting for endogenous matching rates while ignoring the equity motive of taxation.

How labour market policies affect the equilibrium in frictional environments has been studied by Pissarides (1985, 1998), Lockwood and Manning (1993), and Boone and Bovenberg (2002). The present article features job mismatching as an additional dimension while preserving a simple and intuitive tax result. Shimer and Smith (2001) and Blázquez and Jansen (2008) also introduce taxes in presence of equilibrium mismatch. They consider respectively taxes on search intensities, and taxes paid by unemployed workers mainly for an illustrative purpose. Instead, my focus on optimal taxes aims at discussing the implications in terms of redistribution.

The composition externality that results in inefficient laissez-faire equilibria has been identified in various environments featuring heterogeneity and Nash bargaining. The composition externality always leads to theled result that "agents do not aim high enough" at equilibrium. The literature can be classified into three categories according to the nature of agents' decisions. The first category of articles considers job-acceptance strategies and (undirected) search efforts of heterogeneous agents in a random search framework (Lockwood, 1986; Shimer and Smith, 2001; Albrecht and Vroman, 2002; Blázquez and Jansen, 2008). The second category focuses on ex-ante technological or educational investments before matching (Acemoglu and Shimer, 1999; Acemoglu, 2001; Amine and Santos, 2008). These two categories of papers have in common that search is random in a unified labour market. A worker has the same odds of finding a good match or a bad match. This article belongs to a third category in which the labour market is exogenously partitioned and search is directed towards different segments of the labour market (Moscarini, 2001; Charlot and Decreuse, 2005; Uren, 2006; Decreuse, 2008; Estache and Foucart, 2018). This environment is more suitable to emphasise the role of search strategies. In this article, both the quantitative and qualitative margins of search activities are modelled.

Recent works on online job search provide empirical supports for the modelling of search activities in this article. Belot et al. (2018) find that online job seekers adjust the "occupational breadth of search" depending on information they receive. This qualitative dimension of search decisions is present in the theoretical model. Marinescu and Wolthoff (2016) show that workers direct more their search towards job titles than towards wages, in line with the segmentation of the labour market I consider. In my setting, workers direct their search towards different submarkets defined by firm types and not by wages. The model in this article therefore distinguishes from the competitive search framework of Moen (1997), or of Menzio and Shi (2011). The equilibrium concept cannot be straightforwardly defined when wages are posted instead of Nash-bargained.

The first section of this paper defines the theoretical framework and proves the inefficiency of the laissez-faire equilibrium. The second section characterises the optimal tax scheme, and its anti-redistributive feature. Section 3 discusses posted wages, on-the-job search and risk aversion in this framework. The model is then calibrated in a fourth section. Lastly, I conclude.

1 The inefficiency of the laissez-faire

This section introduces the theoretical model. The first-best allocation is defined in the second subsection. In the third subsection, the decentralised equilibrium with Nash bargain is characterised and shown to be inefficient in absence of taxes.

1.1 The environment

Time is continuous. The economy is populated by infinitely-lived risk-neutral heterogeneous workers. There is an exogenous measure of workers ν_i of each type $i \in \mathcal{I}$. A worker can be employed in an industry $j \in \mathcal{J}$ where she produces y_{ij} and receives wage w_{ij}. Otherwise, the worker is unemployed and consumes home production h_i. u_i denotes the measure of i-type unemployed workers. A worker must be employed by a firm to produce. To employ a worker in a particular industry, a firm opens an industry-specific job vacancy, and pays a cost k_j until the vacancy is filled. The distribution of firm types is endogenous. v_j denotes the measure of j-type vacancies. A firm j that is employing a worker i makes flow profits π_{ij}. As the production technology exhibits constant returns to scale, multiple-job firms are not different from a collection of single-job firms. A firm will thus refer to a single job position, filled or vacant. The measure of matches between i-type workers and j-type firms is n_{ij}. Firms and workers discount future incomes at rate r.

Search and matching Search frictions prevent the instantaneous matching of job vacancies and workers. Search activities are on workers' own initiatives. A worker can observe the firms' type but cannot distinguish between firms of a given type. Each industry j constitutes an independent segment of the labour market. Workers direct their search towards particular segments, and they choose how intensively to search for each segment. On each segment j, workers of any type can only meet a j-type firm. The meeting probabilities within each segment are governed by a constant-returns-to-scale function in the tradition of Diamond-Mortensen-Pissarides (see Pissarides, 2000, for an overview). Wages are decide after Nash bargaining once a firm and a worker have met. β denotes the bargaining parameter of workers, with $0 \leq \beta < 1$.

On each segment j, a job seeker multiplies her baseline job-finding rate m_j by a search intensity $s(e)$ where e is a segment-specific search cost. $s : \mathbb{R}^+ \to \mathbb{R}^+$ is twice differentiable, increasing and concave. The search technology has decreasing returns to scale at the segment level: the more a worker searches on a particular segment, the less she multiplies her job-finding rate. When a worker does not exert any search effort, her search intensity is nil, $s(0) = 0$, but the marginal search intensity is infinite, $\lim_{e \to 0} s'(e) = \infty$. The marginal search intensity tends to zero as the effort tends to infinity, $\lim_{e \to \infty} s'(e) = 0$.[1] A search strategy consists in choosing a search effort (expressed in cost units) for each segment of the labour market. For the sake of simplicity, I consider only the case in which same-type workers choose the same search strategy. e_{ij} denotes the search effort of i-type unemployed workers for any segment j in J.

The baseline job-finding rate m_j is endogenously determined by a matching technology. The measure of applicants on the segment j is $\int_{\mathcal{I}} s(e_{ij}) u_i di$, expressed in efficiency units. The measure of new meetings between firms and workers is given by $M\left(\int_I s(e_{ij}) u_i di, v_j\right)$. M is a constant-returns-to-scale Cobb-Douglas function. The elasticity of the meeting function relative to the measure of employers is denoted η, with $0 < \eta < 1$. Define industry-specific market tightness θ_j as the ratio of the employers' mass to the efficient job seekers' mass,

$$\theta_j \equiv \frac{v_j}{\int_{\mathcal{I}} s(e_{ij}) u_i di}. \tag{1}$$

The baseline job-finding rate m_j is a function of market tightness, $m_j = m(\theta_j) \equiv M(1, \theta_j)$. Consequently, the function m is differentiable, strictly increasing and concave from $\mathbb{R}^+$ onto $\mathbb{R}^+$, and $\eta = \frac{\theta_j m'(\theta_j)}{m(\theta_j)}$. A i-type worker exerting an effort e_{ij} on segment j meets a firm at the Poisson rate $s(e_{ij}) m(\theta_j)$. Conversely, a j-type firm meets a i-type worker at the rate

$$\frac{s(e_{ij}) u_i}{\int_{\mathcal{I}} s(e_{i'j}) u_{i'} di'} \frac{m(\theta_j)}{\theta_j}.$$

This expression derives from the equality between the flow of workers i meeting a firm j, and the flow of firms j meeting a worker i. $q(\theta_j) \equiv \frac{m(\theta_j)}{\theta_j}$ is the rate at which a firm j meets a worker irrespective of her type. Once a firm and a worker meet, they match and start producing. The decision of each party to agree or not on matching after meeting, namely the job-acceptance margin, can be ignored. At equilibrium, workers only apply for jobs that are acceptable by the firm. Jobs in industry j exogenously break at rate δ_j.

[1] The model readily accommodates for match-specific search technology, namely s as function of both the effort and the match (i, j). For instance, a same effort e could further improve the job-finding rate for job seekers more productive in the industry. The results for efficiency and optimal taxation are unaffected.

The operator $\mathbb{E}_{i|j}$ is defined by

$$\mathbb{E}_{i|j}A_{ij} \equiv \frac{\int_{\mathcal{I}} A_{ij}s(e_{ij})u_i di}{\int_{\mathcal{I}} s(e_{ij})u_i di}.$$

$\mathbb{E}_{i|j}A_{ij}$ is the expected value of a variable A over the distribution of newly employed workers in industry j. In particular, $\mathbb{E}_{i|j}y_{ij}$ is the mean productivity of new jobs in industry j. Notice i is a mute variable in $\mathbb{E}_{i|j}A_{ij}$, contrary to j. The operator $\mathbb{E}_{i|j}$ is endogenous as it depends on the search strategies and the measures of unemployed workers.

Flows and transitions Denote $\dot{n}_{ij}$ and $\dot{u}_i$ the time derivative of the number of matches and the number of unemployed workers. For an infinitesimal time period Δt, a fraction $s(e_{ij})m(\theta_j)\Delta t$ of i-type unemployed workers gets employed in a j-type firm. Simultaneously, a fraction $\delta_j\Delta_t$ of employed workers becomes unemployed in each sector j. The dynamics of employment and unemployment is defined by the following equations at the limit when Δt tends to zero:

$$\dot{n}_{ij} = s(e_{ij})m(\theta_j)u_i - \delta_j n_{ij}, \tag{2}$$

$$u_i = \nu_i - \int_{\mathcal{J}} n_{ij}dj. \tag{3}$$

Both the benevolent social planner and private agents are constrained by search frictions. The analysis in the paper focuses on the steady state.

Definition 1 *A steady-state allocation is*

- *a distribution of agents* $\mathcal{D} = \{n_{ij}, u_i, v_j\}_{i\in\mathcal{I},j\in\mathcal{J}}$ *such that there exist industry-specific market tightnesses* $\boldsymbol{\theta} = \{\theta_j\}_{j\in\mathcal{J}}$ *and search efforts* $\boldsymbol{e} = \{e_{ij}\}_{i\in\mathcal{I},j\in\mathcal{J}}$ *that satisfy equations* (1), (2) *and* (3) *at the steady state,*
- *a pay-off profile* $\mathcal{P} = \{w_{ij}, \pi_{ij}\}_{i\in\mathcal{I},j\in\mathcal{J}}$ *for workers and firms that are matched.*

An allocation is feasible if and only if total output exceeds vacancy costs and transfers

$$\int_{\mathcal{I}\times\mathcal{J}} (\pi_{ij} + w_{ij})\, n_{ij}didj + \int_{\mathcal{J}} k_j v_j dj \leq \int_{\mathcal{I}\times\mathcal{J}} y_{ij}n_{ij}didj.$$

Importantly, a pair $(\boldsymbol{\theta}, \boldsymbol{e})$ uniquely determines a steady-state distribution $\mathcal{D}$.[2]

[2]Assume $(\boldsymbol{\theta}^1, \boldsymbol{e}^1)$ and $(\boldsymbol{\theta}^2, \boldsymbol{e}^2)$ generate the same distribution $\{n_{ij}, u_i, v_j\}$. Equation (2) implies $s(e^1_{ij})m(\theta^1_j) = s(e^2_{ij})m(\theta^2_j)$. Therefore $\frac{\theta^1_j}{m(\theta^1_j)} = \frac{\theta^2_j}{m(\theta^2_j)}$ from equation (1). Necessarily, $\theta^1_j = \theta^2_j$ and $e^1_{ij} = e^2_{ij}$.

1.2 Efficient allocation

Consider the problem of a benevolent utilitarian social planner who decides i) the number of job vacancies to create, or equivalently market tightnesses $\boldsymbol{\theta}$, and ii) the search strategies $\boldsymbol{e}$. The social planner only cares about efficiency because workers are risk-neutral. The pay-off profile $\mathcal{P}$ has no consequences on social welfare, as long as the feasibility condition is binding. The social planner maximises total discounted net output $\int_0^\infty Y_t \exp(-rt)dt$. Net current-period output is defined by

$$Y_t \equiv \int_{\mathcal{I}\times\mathcal{J}} y_{ij}n_{ij}didj + \int_{\mathcal{I}} h_i u_i di - \int_{\mathcal{J}} k_j v_j dj - \int_{\mathcal{I}} e_{ij}u_i di.$$

Net output is the sum of four terms: employment production, home production, the vacancy costs and the search costs. Optimisation is subject to constraints (1), (2) and (3).

The next lemma characterises the efficient allocation in terms of the value of unemployment. It is the equivalent to Proposition 1 of Acemoglu and Shimer (1999), here in a two-sided heterogeneity framework. Following these authors, ψ_i denotes the social flow value of i-type unemployment. One more unemployed worker of type i increases steady-state output by ψ_i/r. ψ_i is directly comparable to a productivity level and can be interpreted as the opportunity cost of being employed. The dynamic planner's problem with constraints reduces to a simple maximisation program.

Lemma 1 *The planner's problem defines the relative values of unemployment,* $\boldsymbol{\psi} = \{\psi_i\}_{i\in\mathcal{I}}$, *as*

$$\psi_i = h_i + \int_{\mathcal{J}} s(e_{ij})m(\theta_j)\left[\frac{y_{ij}-\psi_i}{r+\delta_j}\right]dj - \int_{\mathcal{J}} k_j s(e_{ij})\theta_j dj - \int_{\mathcal{J}} e_{ij}dj. \tag{4}$$

Define $\Psi_i(\boldsymbol{\theta}, \boldsymbol{e}, \psi_i)$ *as the right-hand side of this equation. The planner's problem is equivalent to maximising the mean value of unemployment,* $\int_{\mathcal{I}} \Psi_i(\boldsymbol{\theta}, \boldsymbol{e}, \psi_i)u_i di$, *with respect to* $\boldsymbol{\theta}$ *and* $\boldsymbol{e}$.

A proof is given in appendix A. The value of unemployment accounts for home production (first term). The second term discounts the perspectives of future employment. At a rate $s(e_{ij})m(\theta_j)$, the unemployed worker finds a job yielding the present-discounted net gain $\frac{y_{ij}-\psi_i}{r+\delta_j}$. Jobs are more socially valuable when the discount rate and the separation rate are lower because individuals are more patient and jobs last longer. The two negative terms represent the vacancy cost per unemployed worker and the individual search cost.

I apply Lemma 1 and derive the first-order conditions for θ_j and e_{ij}. A classical trade-off arises when fixing market tightness. On the one hand, workers match and start producing more rapidly when the labour market is tight for firms ("thick-market externality"). The second term at the right-hand side of equation (4) is thus increasing in θ_j. On the other hand, firms wait longer before meeting a worker and so they have to pay a higher expected vacancy cost ("congestion externality"). The third term in equation at the right-hand side is thus decreasing. The first-order condition relative to market tightness θ_j can be written as the balance between these negative and positive externalities at the margin,

$$\frac{k_j}{q(\theta_j)} = \eta \mathbb{E}_{i|j}\left[\frac{y_{ij} - \psi_i}{r + \delta_j}\right], \qquad \text{or } \theta_j = 0. \tag{5}$$

The expected vacancy cost on the left-hand side must be equal to the share η of the expected match surplus, defined by the expected job productivity net of the unemployment value. If jobs do not produce a positive surplus on expectation, $\mathbb{E}_{i|j}[y_{ij} - \psi_i] < 0$, then no vacancy is opened, $\theta_j = 0$.

The efficient search effort equalises the marginal gain from searching to the marginal cost,

$$s'(e_{ij})m(\theta_j)\left(\frac{y_{ij} - \psi_i}{r + \delta_j} - \frac{k_j}{q(\theta_j)}\right) = 1, \qquad \text{or } e_{ij} = 0. \tag{6}$$

The term in parentheses is the net value of a match: the expected production net of the value of unemployment and the vacancy cost. Jobs whose net value is negative are not prospected. As long as the net value of a match is positive, searching is efficient because the marginal gain from searching will be infinite at a zero level of efforts. The marginal cost is 1 for any segment of the labour market, therefore the marginal gains from searching equalise across prospected segments. The optimal search strategy e_{ij} is decreasing in y_{ij}. In other words, workers have to search more for jobs whose productivity is higher.

Definition 2 *An efficient (or first-best) steady-state allocation is characterised by values of an unemployed worker* $\boldsymbol{\psi}$*, market tightnesses* $\boldsymbol{\theta}$ *and search efforts* $\mathbf{e}$ *fulfilling* (4), (5) *and* (6).

For the same reasons as Shimer and Smith (2001), some mismatch may be efficient in the presence of search frictions. By being more selective, workers pay more search costs and firms wait longer before matching. It may be efficient for a worker to apply to jobs that are not their best fit in order to increase the job-filling rate of firms.

1.3 Decentralised equilibrium

1.3.1 The Bellman equations

Consider the decisions of workers and firms regarding search activities and job openings. Let W_{ij} be the worker present-discounted value of a (i, j)-match. U_i is defined as the asset value of being unemployed for a worker of type i. When a worker is unemployed, she consumes home production and incurs search costs. She obtains a (i,j)-type job at the rate $s(e_{ij})m(\theta_j)$ and makes a capital gain $W_{ij}-U_i$. The asset value of unemployment is accordingly defined by

$$rU_i = h_i + \int_{\mathcal{J}} s(e_{ij})m(\theta_j)\left[W_{ij}-U_i\right]dj - \int_{\mathcal{J}} e_{ij}dj. \tag{7}$$

An employee earns the net-of-tax wage w_{ij}. She may return to the pool of unemployment if her match breaks, which occurs at a rate δ_j. The asset value of a match satisfies

$$rW_{ij} = w_{ij} + \delta_j\left(U_i - W_{ij}\right). \tag{8}$$

When matched, a worker renounces the flow value of unemployment rU_i until the job destruction. By definition, $\psi_i \equiv rU_i$ in the decentralised equilibrium. Combining equations (7) and (8) provides an expression for the value of an unemployed worker:

$$\psi_i = h_i + \int_{\mathcal{J}} s(e_{ij})m(\theta_j)\left[\frac{w_{ij}-\psi_i}{r+\delta_j}\right]dj - \int_{\mathcal{J}} e_{ij}dj. \tag{9}$$

The value of unemployment defined here differs from the social value defined in (4) in two respects. First, workers compare the wage to the value of unemployment $(w_{ij}-\psi_i)$, whereas the planner compares the match output to the value of unemployment $(y_{ij}-\psi_i)$. Second, workers do not internalise the vacancy cost.

Workers choose the search strategy that maximises their return in (9). The search effort then satisfies

$$s'(e_{ij})m(\theta_j)\left(\frac{w_{ij}-\psi_i}{r+\delta_j}\right) = 1, \qquad \text{or } e_{ij}=0. \tag{10}$$

When the labour income w_{ij} is lower than the returns to unemployment ψ_i, the unemployed workers do not exert any search effort.[3] If the value of a match from a worker's perspective is high enough, job seekers are willing to search until the marginal benefit equals 1.

From the perspective of the firms, J_{ij} denotes the firm's asset value of a (i,j)-type match. Prior to meeting a worker, the expected value is $\mathbb{E}_{i|j}J_{ij}$. Firms create jobs until reaching zero profit at the steady state. The no-arbitrage condition for free entry writes as the equality between the expected cost of holding a vacancy and the expected value of filling a vacancy. If the expected value is negative, no firm enters the market. The

[3] Actually, workers do not exert any effort either if $\pi_{ij}<0$ because the match would be rejected by the firm. It is implicitly assumed that $\pi_{ij}>0$ when $w_{ij}>\psi_i$ for the exogenous wage profile. Nash-bargained wages satisfy this condition.

condition can be expressed

$$\frac{k_j}{q(\theta_j)} = \mathbb{E}_{i|j} J_{ij}, \qquad \text{or } \theta_j = 0. \tag{11}$$

From the firm's side, the *ex-post* surplus of a (i, j)-match, J_{ij}, must be distinguished from the *ex-ante* surplus, $J_{ij} - \frac{k_j}{q(\theta_j)}$. The no-arbitrage condition is equivalent to a null ex-ante surplus in expectation. When the employer meets a potential employee, she accepts the match as long as the ex-post surplus is positive, $J_{ij} > 0$. The vacancy cost is sunk before the meeting, and so it is not accounted for in the job-acceptance decision. Following the literature, surplus will refer to ex-post surplus. The ex-ante surplus will always be specified as such. Define the joint surplus of a job, $\Omega_{ij} = W_{ij} - U_i + J_{ij}$. A match yields a net flow profit π_{ij} to the employer, with a risk to be broken at a rate δ_j,

$$rJ_{ij} = \pi_{ij} - \delta_j J_{ij}. \tag{12}$$

From equations (11) and (12), the total expected vacancy cost is equal to the present-discounted value of expected profits:

$$\frac{k_j}{q(\theta_j)} = \frac{\mathbb{E}_{i|j} \pi_{ij}}{r + \delta_j}, \qquad \text{or } \theta_j = 0. \tag{13}$$

Higher profits attract more firms so market tightness increases with expected profits.

1.3.2 Optimal pay-off profiles

Consider the problem of a benevolent utilitarian social planner who let agents determine themselves market tightnesses $\boldsymbol{\theta}$ and search efforts $\boldsymbol{e}$, but who controls pay-offs π_{ij} and w_{ij} for any $i \in \mathcal{I}$ and $j \in \mathcal{J}$. Can the social planner decentralise the first-best allocation with appropriate transfers? I prove here that it can.

A pay-off rule is defined as endogenous when wages w_{ij} and profits π_{ij} depends on equilibrium variables $\boldsymbol{\psi}$, $\boldsymbol{\theta}$ and $\boldsymbol{e}$. Denote it $\mathcal{P}(\boldsymbol{\psi}, \boldsymbol{\theta}, \boldsymbol{e})$. The problem of the social planner consists in finding an appropriate pay-off rule, possibly endogenous.

Definition 3 *A decentralised equilibrium with a pay-off rule $\mathcal{P}(\boldsymbol{\psi}, \boldsymbol{\theta}, \boldsymbol{e})$ is characterised by values of an unemployed worker $\boldsymbol{\psi}$, market tightness $\boldsymbol{\theta}$, and search efforts $\boldsymbol{e}$ fulfilling equations* (9), (10) *and* (13).

A comparison with Definition 2 gives sufficient conditions for any decentralised equilibrium to be efficient.

Proposition 1 *There exist pay-off rules such that any decentralised equilibrium is efficient. Profits must be such that firms obtain a share* η *of the match surplus on expectation,*

$$\mathbb{E}_{i|j}\pi_{ij} = \eta\mathbb{E}_{i|j}[y_{ij} - \psi_i], \qquad \textit{when } \theta_j > 0. \tag{14}$$

The discounted income a worker receives form being employed in a particular industry must be equal to the discounted output of the job net of the vacancy cost,

$$\frac{w_{ij}}{r+\delta_j} = \frac{y_{ij}}{r+\delta_j} - \frac{k_j}{q(\theta_j)}, \qquad \textit{when } e_{ij} > 0. \tag{15}$$

These conditions are necessary and sufficient.

Proof. These conditions are sufficient because they lead equations (9), (10) and (13) to be equivalent to equations (4), (5) and (6). I show that the conditions are necessary. Suppose a decentralised equilibrium defined by $(\boldsymbol{\psi}, \boldsymbol{\theta}, \boldsymbol{e})$ is efficient. Then, there exists $\boldsymbol{\psi}^*$ fulfilling (4) such that

$$\mathbb{E}_{i|j}\pi_{ij} = \eta\mathbb{E}_{i|j}[y_{ij} - \psi_i^*], \qquad \text{when } \theta_j > 0,$$
$$\frac{w_{ij} - \psi_i}{r+\delta_j} = \frac{y_{ij} - \psi_i^*}{r+\delta_j} - \frac{k_j}{q(\theta_j)}, \qquad \text{when } e_{ij} > 0.$$

Equations (4) and (9) thus lead to $\psi_i - \psi_i^* = 0$, and so conditions (14) and (15). ■

Expected profits provide the right incentives for firms to create jobs. The first condition can be written in terms of surplus, $\mathbb{E}_{i|j}J_{ij} = \eta\mathbb{E}_{i|j}\Omega_{ij}$. The sharing of the expected surplus accounts for the ability of each side to create positive versus negative search externalities, depending on the meeting function. When the elasticity of the matching function η is high, firms are more efficient than workers in the search process and consequently deserve higher gains from matching. The congestion externality (negative) they impose on the other firms, which want to fill their vacancy too, is lower and the thick-market (positive) externality on job seekers is higher. Symmetrically, when η is low, job seekers produce better externalities and so they should receive a higher share of the surplus.

The wage profile provides the right incentives for job seekers to search efficiently. When exerting search efforts, workers value jobs according to the worker's surplus. The higher this surplus, the higher is the search effort. The second condition makes the worker receives as a surplus the total surplus of the match net of the vacancy cost, $W_{ij} - U_i = \Omega_{ij} - \frac{k_j}{q(\theta_j)}$. This condition makes workers fully internalise the value of each job when searching. This equation is always true on expectation to satisfy the entry condition of firms (11). In condition (15), the slope of the wage curve with respect to productivity, $\frac{\partial w_{ij}}{\partial y_{ij}}$, must be equal to one.

Proposition 1 is a prerequisite to investigate the efficient decentralised bargaining of wages and profits. It easily generalises to industry-specific matching functions and search technology, by denoting the industry-specific elasticity η_j. This proposition does not hold when workers are risk-averse, even with unemployment benefits. With risk-neutral agents, the social planner can choose any pay-off rule satisfying Proposition 1 to provide the right incentives to agents. When risk aversion is introduced, the efficient allocation requires workers to be paid the same. The social planner looses pay-offs as a tool to incentivise agents.

1.3.3 Nash bargain

Consider now that agents determine pay-offs through Nash bargain. Agents determine market tightnesses $\boldsymbol{\theta}$ and search efforts $\boldsymbol{e}$, but also wages and profits. I show that no laissez-faire equilibrium can decentralise an efficient allocation.

The government taxes an amount t_{ij} from each (i,j)-job. With w_{ij} as the *after-tax* wage, flow profits write

$$\pi_{ij} \equiv y_{ij} - w_{ij} - t_{ij}.$$

Whether the fiscal authority asks workers or firms to pay taxes on jobs leads to the same wage setting with Nash bargain. There is no difference in tax incidence. The tax scheme is characterised by the minimum number of parameters to achieve efficiency, namely a proportional wage tax and match-specific lump-sum taxes. Define $t_{ij} = \tau^w.w_{ij} + \tau^f_{ij}$. It can be surprising to use a wage tax given that there is a unique after-tax wage per match (i,j). A marginal increase in the tax rate $\Delta\tau^w$ is actually not equivalent to an increase in the lump-sum tax by $\Delta\tau^f = \Delta\tau^w w$. The first policy modifies the wage setting under Nash bargain contrary to the second. I impose $1+\tau^w > 0$ so that profits are decreasing with wages and the Nash-bargaining objective is concave. The laissez-faire is the case $t_{ij} = 0$ and $\tau_w = 0$.

Wages are solutions of the following maximisation:

$$\max_{w_{ij}} \; (W_{ij} - U_i)^\beta \, J_{ij}^{1-\beta} \text{ s.t. (8) and (12).}$$

The pay-off rule $\mathcal{P}^{tax}(\boldsymbol{\psi}, \boldsymbol{\theta}, \boldsymbol{e})$ under the tax scheme is given by

$$\pi_{ij} = (1-\tilde{\beta})(y_{ij} - t_{ij} - \psi_i) \qquad \text{and} \qquad w_{ij} = \tilde{\beta}(y_{ij} - t_{ij}) + (1-\tilde{\beta})\psi_i. \tag{16}$$

with $\tilde{\beta} = \frac{\beta}{\beta + (1-\beta)(1+\tau^w)}$. Taxes affect private behaviours through two channels.

First, the wage tax affects the worker's share of the surplus $\tilde{\beta}$. If the tax rate is positive, firms incur a higher cost of labour for a constant after-tax wage. An employee then has to bargain more aggressively to obtain the same wage as in a laissez-faire equilibrium. $\tilde{\beta}$

is thus decreasing in τ^w. The effect of payroll taxes on wage bargaining is emphasised by Boone and Bovenberg (2002); Lockwood and Manning (1993); Pissarides (1985, 1998). Second, taxes modify the match surplus to be shared between the firm and the employee, $\frac{y_{ij}-t_{ij}-\psi_i}{r+\delta_j}$. By taxing and subsidising jobs, the fiscal authority impacts the attractiveness of some matches.

A decentralised equilibrium with bargained wages and taxes satisfies Definition 3 with the pay-off rule $\mathcal{P}^{Tax}$.

Lemma 2 *At a decentralised equilibrium with Nash bargain and the pay-off rule* $\mathcal{P}^{Tax}(\boldsymbol{\psi}, \boldsymbol{\theta}, \boldsymbol{e})$*, profits satisfy*

$$\mathbb{E}_{i|j}\pi_{ij} = (1-\tilde{\beta})\mathbb{E}_{i|j}[y_{ij} - t_{ij} - \psi_i], \tag{17}$$

expected discounted incomes satisfy

$$\frac{\mathbb{E}_{i|j}w_{ij}}{r+\delta_j} = \frac{\mathbb{E}_{i|j}[y_{ij} - t_{ij}]}{r+\delta_j} - \frac{k_j}{q(\theta_j)}, \tag{18}$$

and relative wages satisfy

$$w_{ij} - \mathbb{E}_{i|j}w_{ij} = \tilde{\beta}\left(y_{ij} - t_{ij} - \mathbb{E}_{i|j}[y_{ij} - t_{ij}]\right) + (1-\tilde{\beta})\left(\psi_i - \mathbb{E}_{i|j}\psi_i\right). \tag{19}$$

These expressions of expected profits, expected wages and relative wages are comparable to the conditions in Proposition 1.

Inefficient surplus sharing Firms obtain on expectation a share $1-\tilde{\beta}$ of the match surplus. At the laissez-faire equilibrium, equation (17) becomes equivalent to equation (14) in Proposition 1 only when the Hosios-Pissarides condition applies: $1-\beta = \eta$. A graphical illustration of this inefficiency is provided by Figure 1. Optimal market tightness is given by the maximisation defined in Lemma 1. This maximisation problem can be rewritten as the maximisation of the average returns to unemployment constrained by the zero-profit condition to determine market tightnesses and mean wages:

$$\max_{(\boldsymbol{\theta}, \bar{\boldsymbol{w}})} \int_{\mathcal{I}} h_i u_i di + \int_{\mathcal{I}\times\mathcal{J}} s(e_{ij}) m(\theta_j) \left[\frac{\bar{w}_j - \mathbb{E}_{i|j}\psi_i}{r+\delta_j}\right] u_i di dj - \int_{\mathcal{I}\times\mathcal{J}} e_{ij} u_i di dj \tag{WU}$$

$$\text{s.t. } \frac{k_j}{q(\theta_j)} = \frac{\mathbb{E}_{i|j}y_{ij} - \bar{w}_j}{r+\delta_j} \tag{ZP}$$

For a given industry j, the surplus sharing within each sector can be illustrated in a *market tightness-mean wage* plan $(\theta_j, \mathbb{E}_{i|j}w_{ij})$, keeping other variables constant. The zero-profit condition (ZP) is a fixed decreasing curve in this plane. The objective function (WU) defines a set of decreasing isoutility curves that do not cross each other. The isoutility curves share the same horizontal asymptote at level $\mathbb{E}_{i|j}\psi_i$. The graphical transcription of the constrained maximisation consists in finding the isoutility curve that i) corresponds to the highest workers' utility, and ii) still crosses the zero-profit condition. The isoutility curve is therefore tangent to the zero-profit curve at optimal market tightness. This condition is met only if the surplus is shared according to the weights η and $1-\eta$ as the curve (WU*) shows. In the laissez-faire equilibrium, the wage is determined by the worker's share of the surplus β. Figure 1 illustrates the case $\beta < 1-\eta$. Workers' utility can be increased by bargaining a higher mean wage and by reducing market tightness, keeping profits unchanged.[4]

The Hosios-Pissarides condition, $\beta < 1-\eta$, is standard result in the Diamond-Mortensen-Pissarides framework with homogeneous agents. It requires the surplus sharing to balance the search externalities generated by firms and workers. When firms are efficient in the search process, equivalently the elasticity η is high, they should receive a higher share of the surplus $1-\beta$.

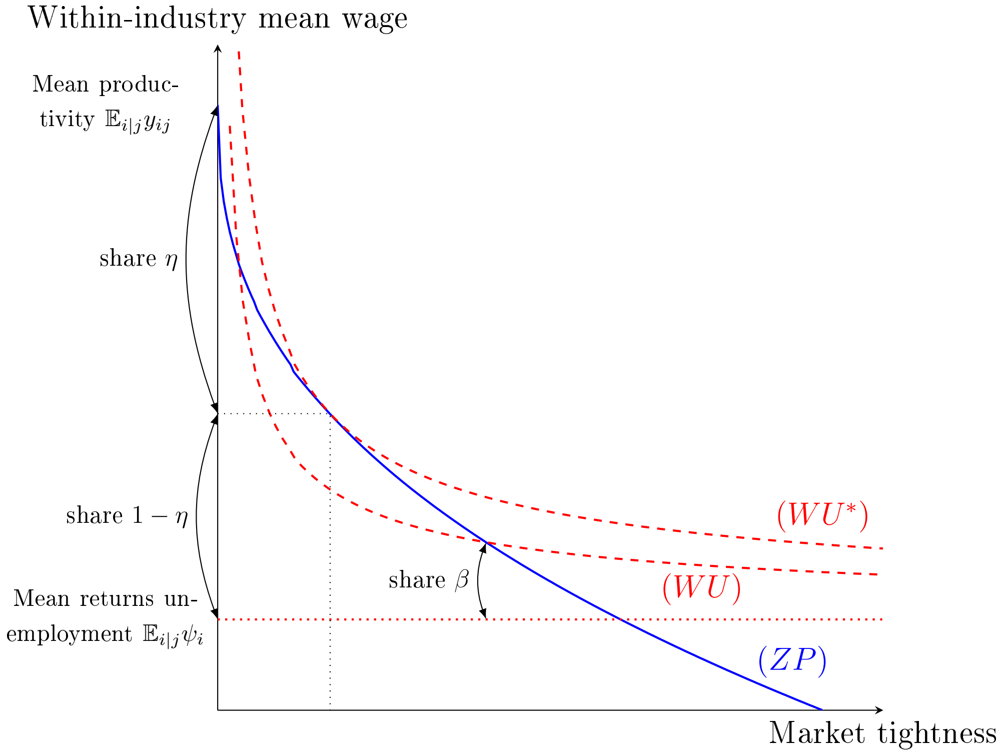

Figure 1: Efficient vs. laissez-faire surplus sharing

Note: the X-axis represents market tightness θ_j and the Y-axis the mean wage $\mathbb{E}_{i|j} w_{ij}$. The blue curves (ZP) correspond to the zero-profit condition for firms. The red dashed curves are isoutility curves on which the mean returns to unemployment are constant.

Inefficient search strategy The expression of expected wages (18) derives from the free entry condition (13). At the laissez-faire equilibrium, this equation is compatible with an efficient wage rule defined by equation (15) in Proposition 1. An inefficiency

[4] Moen (1997) draws similar graphs in the competitive search framework.

arises from the third equation in Lemma 2. Equation (19) characterises the deviation of wages from within-industry mean wages. From Proposition 1, any deviation of job productivity from the within-industry mean must result in the same deviation of wages, $w_{ij} - \mathbb{E}_{i|j} w_{ij} = y_{ij} - \mathbb{E}_{i|j} y_{ij}$. At the laissez-faire equilibrium, wages are set optimally only if workers have full bargaining power, $\beta = 1$.

Figure 2 illustrates the gap between the efficient wage setting given in Proposition 1 and the decentralised wage at the laissez-faire equilibrium. Consider the plane $(\Omega_{ij}, W_{ij} - U_i)$ when market tightnesses and values of unemployment are fixed. The efficient wage setting in equation (15) defines a line (OW) of equation $W_{ij} - U_i = \Omega_{ij} - \frac{k_j}{q(\theta_j)}$. The right-hand side is the ex-ante surplus of a job. Equation (18) in Lemma 2 states that the decentralised equilibrium wage curve (EW) crosses the curve (OW) at the point $(\mathbb{E}_{i|j}\Omega_{ij}, \mathbb{E}_{i|j}[W_{ij} - U_i])$. In the laissez-faire equilibrium $\tilde{\beta} = \beta$, equation (19) characterises (EW) in terms of surpluses: $W_{ij} - U_i - E_{i|j}[W_{ij} - U_i] = \beta(\Omega_{ij} - E_{i|j}\Omega_{ij})$.

When a worker generates a higher surplus than the average worker in the industry, for instance $\Delta\Omega = \Omega_{ij} - E_{i|j}\Omega_{ij}$, the surplus she gets is lower than $\Delta\Omega$ due to the sharing with the employer. On the one hand, the Nash bargain prevents workers from accruing the full benefits of an increase in productivity. On the other hand, workers fully incur the cost of being selective, as an explicit search cost or as an opportunity cost to leave unemployment quicker. As long as $\beta < 1$, workers therefore undervalue the gains from being in a productive match compared to the social planner. They devote too much effort in searching for low-surplus jobs to the detriment of high-surplus jobs, market tightness and values of unemployment being fixed.

A range of low-productive jobs are prospected at equilibrium but should not be so. These jobs produce a positive ex-post surplus: the output is higher than the value of unemployment. Both the worker and the firm benefit from matching and can bargain a wage. These jobs still have a negative ex-ante surplus: the output is not high enough to compensate both the value of unemployment and the vacancy cost. Workers, knowingly, apply to these jobs. Even if firms have interest in rejecting such low-quality matches ex-ante, they lack a commitment device to make rejection a credible threat. Workers internalise the composition externality only when they obtain the full surplus of the job net of the vacancy cost. When $\beta = 1$, the curves (EW) and (OW) overlap.

Notice the second inefficiency, due to the composition externality, is close to the holdup problem as formulated by Grout (1984). In holdup problems, workers (or firms) make an investment before matching on the labour market. As the cost of this investment is sunk before any meeting, workers (firms) under-invest for as long as they do not have full bargaining power. Search strategies can thus be compared to worker investments. The inefficiency due to the composition externality, however, is singular as it arises only with endogenous job creation. If the number of firms or jobs were fixed, the composition

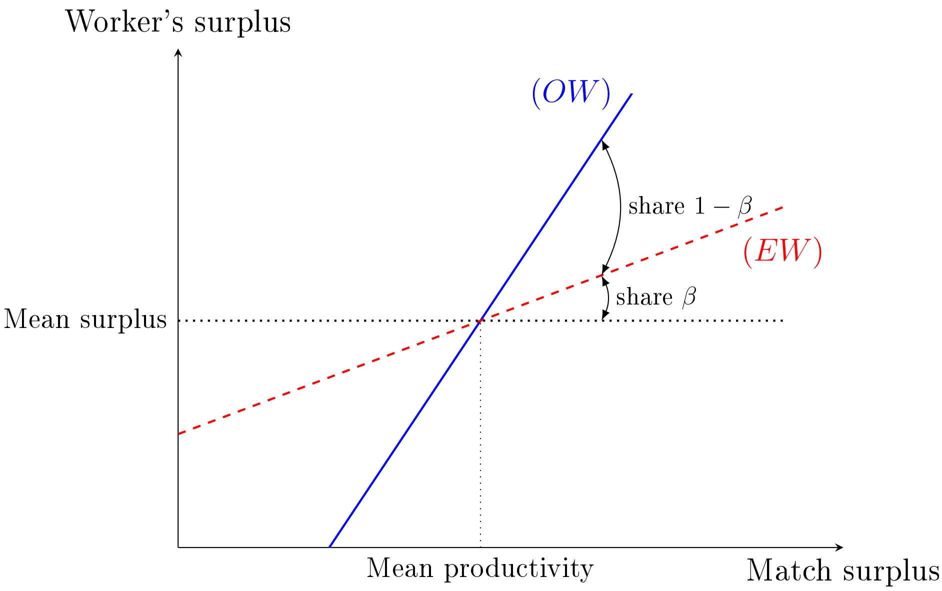

Figure 2: Efficient vs. laissez-faire search strategy

Note: the X-axis represents the (ex-post) match surplus Ω_{ij} and the Y-axis the associated worker's surplus $W_{ij} - U_i$. The continuous blue curve (OW) corresponds to the optimal wage profile. The red dashed curve (EW) corresponds to the decentralised wage setting in the laissez-faire.

externality would vanish.[5]

Conditions (14) and (15) are necessary for an equilibrium to be efficient. Combined to Lemma 2, these conditions require the bargainging parameter to satisfy two incompatible equations: the Hosios-Pissarides condition, $\beta = 1 - \eta$, and full bargaining to workers, $\beta = 1$.

Proposition 2 *There is no bargaining power β nor elasticity of the matching function η such that a segmented laissez-faire equilibrium is optimal. In particular, the Hosios-Pissarides condition, $\eta = 1 - \beta$, does not restore the efficient allocation.*

I define a segmented equilibrium as an equilibrium where at least two different workers exert search efforts on the same active submarket. More formally, there exists $j \in \mathcal{J}$ such that $\theta_j > 0$. There also exist $i, i' \in \mathcal{I}$ such that search efforts are exerted $e_{ij} > 0$, $e_{i'j} > 0$, and the flow surplus is different, $y_{ij} - \psi_i \neq y_{i'j} - \psi_{i'}$. If this condition is not satisfied, the equilibrium is degenerated in the sense that workers generate the exact same surplus on each segment. For degenerated equilibria, the composition externality disappears and so the Hosios-Pissarides condition is sufficient to restore efficiency.

The laissez-faire allocation can only be *asymptotically* efficient when the elasticity η tends to 0 and the bargaining power tends to 1. In that case, workers generate very few search externalities within the matching function. The two objectives of the bargaining

[5]See Acemoglu (1996, 2001); Acemoglu and Shimer (1999); Davis (2001); Masters (1998) for holdup problems with frictional labour markets.

power, namely full bargaining power and the Hosios-Pissarides condition, converge in that case.

2 Decentralising the optimal allocation

I now determine the optimal tax scheme and its implications for redistribution. As for Shimer and Smith (2001) and Blázquez and Jansen (2008), optimal taxes shed particular light on the inefficiency of the equilibrium. Optimal taxes constitute a metric to measure the gap between the laissez-faire equilibrium and the social optimum.

2.1 Optimal taxes

The government can choose the tax parameters that provide the right incentives to workers and firms as in Proposition 1. Consider the equations in Lemma 2. First, the expected tax burden should be null within each industry, $\mathbb{E}_{i|j}t_{ij} = 0$, for expected wages to satisfy (15). Subsequently, an augmented Hosios-Pissarides condition restores efficient profits in (14), $1 - \tilde{\beta} = \eta$. Lastly, the level of taxes t_{ij} is determined by insuring (15) for each match. The level of taxes to the within-industry mean level is proportional to the job surplus relative to its within-industry mean, $t_{ij} - \mathbb{E}_{i|j}t_{ij} = -\frac{1-\tilde{\beta}}{\tilde{\beta}}(y_{ij} - \psi_i - \mathbb{E}_{i|j}[y_{ij} - \psi_i])$. These conditions are sufficient and necessary.

Proposition 3 *A decentralised equilibrium with Nash bargain and taxes is efficient if and only if*

- *the wage tax rate satisfies*

$$\tau^{w*} = -\frac{1-\beta-\eta}{(1-\beta)(1-\eta)}, \tag{20}$$

- *the fiscal authority debits the following amount for a (i,j)-type job:*

$$t^*_{ij} = -\frac{\eta}{1-\eta}(y_{ij} - \psi_i - \mathbb{E}_{i|j}[y_{ij} - \psi_i]), \tag{21}$$

where values of unemployment and the expectation operators are those prevailing at the social optimum.

Notice the tax scheme is feasible, $1 + \tau^{w*} > 0$, and the government does not make any loss, $\int_{\mathcal{I}\times\mathcal{J}} t^*_{ij} n_{ij} di dj = 0$.

The optimal tax scheme is remarkable on three aspects. First, it exhibits only a limited number of parameters: the bargaining power β, the matching elasticity η and the flow match surpluses $y_{ij} - \psi_i$. The optimal tax scheme could be implemented with a proportional wage tax, a proportional tax on surplus and an industry-specific lump-sum tax.[6]

Second, the government can achieve the first-best allocation *without* observing agents' actions, namely the search efforts and market tightnesses. The government is only required to observe types i and j, and to know the efficient allocation. This result contrasts with the benchmark moral hazard problem for instance.

Lastly, the interpretation of the optimal tax scheme is intuitive. Even though the model accounts for two-sided heterogeneity, the formula for τ^{w*} is the same as in the homogenous case of Boone and Bovenberg (2002). The optimal wage tax restores an efficient surplus sharing. The level of taxes t^*_{ij} is determined to redistribute from low-surplus jobs to high-surplus jobs within each industry. By taxing low-surplus and subsidizing high-surplus jobs within each industry, the government gives incentives for workers to search less for low-surplus and more for high-surplus jobs. Graphically, this taxation increases the slope of the wage curve (EW) on Figure 2.

Two jobs generating the same surplus in two different industries are in general taxed differently. Taxation indeed depends on the distance between the match surplus and the within-industry mean surplus.

The optimal tax scheme can be generalised to the case in which the matching elasticity and the bargaining power are industry-specific, η_j and β_j. The goverment must then use an industry-specific wage tax, τ^{w*}_j, defined by the same formula as (20). I show in appendix B that more degrees of freedom in the tax scheme, for instance non-linear wage taxation, still result in the same optimal taxation formula.

2.2 Anti-redistributive policy

The optimal policy consists in redistribution from low to high-surplus jobs within each industry. The optimal policy does not necessarily go against equity motives because high-surplus jobs may not always be high-productivity jobs. I define *anti-redistributive* a policy that taxes low-productive jobs and subsidises high-productive jobs.

Under which conditions do the optimal policy is anti-redistributive within each industry? In other words, when does $y_{ij} - \psi_i < \mathbb{E}_{i|j}[y_{ij} - \psi_i]$ implies $y_{ij} < \mathbb{E}_{i|j} y_{ij}$? Jobs generate a high surplus because productivity y_{ij} is high, or because the outside option of the worker ψ_i is low. The optimal policy may thus not be anti-redistributive within each industry. I provide two examples under which the optimal policy is not anti-redistributive.

Assume in a first example that workers have negligible differences in productivity y_{ij} but large differences in home production h_i. The ranking of jobs in terms of surplus would mainly reflect heterogeneity in home production rather than heterogeneity in job productivity. Workers in high-surplus jobs are those with low home production. The

[6]This paper ignores the practical issue of observing and taxing job surplus.

optimal policy does not imply anti-redistribution.

Heterogeneity regarding non-market attributes is not the only reason for a different ranking of surpluses and productivity. Assume, in a second example, that i-type workers and i'-type workers have the same productivity except in some industries for which i-type workers are strictly more productive: $y_{ij} = y_{i'j}$ for $j \in \bar{\mathcal{J}}$, and $y_{ij} > y_{i'j}$ for $j \in \mathcal{J}\backslash\bar{\mathcal{J}}$. Workers of type i have weak absolute advantages. Assume also that home production is the same $h_i = h_{i'}$. Consequently, the value of unemployment is higher for i-type workers, $\psi_i > \psi_{i'}$, because they expect higher returns from matching. Workers of type i' therefore generate a higher surplus in any industry $j \in \bar{\mathcal{J}}$ than workers of type i. i'-type workers will pay less taxes in industries $j \in \bar{\mathcal{J}}$ than i-type workers. This example shows that absolute advantages does not guarantee anti-redistribution within segments.

There is no obvious sufficient condition for an optimal policy to be anti-redistributive. There are, however, three limit cases in which the optimal policy is always anti-redistributive within each industry. In the three cases are supposed to have identical home production h.

Case 1: absolute advantages and homogeneous firms Consider homogeneous firms. Production only depends on the worker's type, $y_{ij} = y_i$. The number of firm types simply defines the number of submarkets. Assume workers play symmetric strategy $e_{ij} = e_{ij'}$ for any j and j'. Everything is now constant in j and the firms' index can be dropped. Define $J = \int_{\mathcal{J}} dj$.

Equation (4) becomes

$$\psi_i = h + Js(e_i)m(\theta)\left(\frac{y_i - \psi_i}{r+\delta} - \frac{k}{q(\theta})\right) - Je_i \tag{22}$$

I prove by contradiction that $y_i - \psi_i < y_{i'} - \psi_{i'}$ is equivalent to $y_i < y_{i'}$. Suppose $y_i > y_{i'}$ and $y_i - \psi_i < y_{i'} - \psi_{i'}$. e_i maximises the left-hand side of this equation. $y_i - \psi_i < y_{i'} - \psi_{i'}$ implies

$$s(e_i)m(\theta)\left(\frac{y_i - \psi_i}{r+\delta} - \frac{k}{q(\theta}\right) - e_i > s(e_{i'})m(\theta)\left(\frac{y_{i'} - \psi_{i'}}{r+\delta} - \frac{k}{q(\theta}\right) - e_{i'}.$$

Therefore $\psi_i < \psi_{i'}$. Using $y_i > y_{i'}$, we obtain the contradiction $y_i - \psi_i > y_{i'} - \psi_{i'}$.

The more productive a worker, the less taxes she will pay according to the optimal tax formula (21). The optimal taxation is not only anti-redistributive within industry but also anti-redistributive overall due to symmetry across industry.

This result holds if home productivity h_i is inversely ordered to market productivity y_i. This heterogeneity reinforces anti-redistribution because high-productive workers create an even larger match surplus from being less productive at home.

Case 2: symmetric comparative advantages *à la* Salop (1979) Consider the type sets $\mathcal{I}$ and $\mathcal{J}$ to be isomorphic to the unit circle. Productivity y_{ij} only depends negatively on the (shortest) arc-distance $\widehat{ij}$ between i and j on the unit circle, $y_{ij} = y_{\widehat{ij}}$.[7] The distribution of workers across types is uniform, ν_i is constant. The cost of vacancy k_j and the job destruction rate δ_j are also constant. The dimensionality of two-sided heterogeneity reduces to one. Absolute advantages are removed. Although two workers may have different types, they cannot be ranked by skills or abilities. Workers merely differ in the industries in which they are productive. As in the previous case, I focus on symmetric equilibria for which θ_j is constant. This environment is adopted by Marimon and Zilibotti (1999) and Decreuse (2008).

Equation (4) becomes

$$\psi = h + \int_0^{2\pi} s(e_x)m(\theta)\left[\frac{y_x - \psi}{r+\delta}\right]dx - \int_0^{2\pi} ks(e_x)\theta dx - \int_0^{2\pi} e_x dx. \tag{23}$$

Two different workers of type i and i' do not exert the same search effort for a given industry j. They do, however, exert the same effort for the same arc-distance. If $x = \widehat{ij} = \widehat{i'j'}$, i-type workers search for a job in industry j with the same effort as i'-type workers search for a job in industry j'. Despite differences in productivity, workers have the same value of unemployment. The level of taxes (21), which only depends on the arc-distance, simplifies to

$$t_x = -\frac{\eta}{1-\eta}(y_x - \mathbb{E}_x y_x). \tag{24}$$

The optimal tax scheme is thus anti-redistributive not only within industries, but overall. Employed workers pay less taxes as they get closer to their employer on the unit circle.

Decentralising the first-best allocation is easier for the social planner in that case. The government can implement the optimal tax scheme with a wage tax, a production tax and a lump-sum component. The tax rates only depends on the bargaining power and the matching elasticity. The wage tax τ^w is still given by (20), $\tau^y = -\frac{\eta}{1-\eta}$ and the lump sum τ^f makes sure to achieve the budget constraint $\mathbb{E}_x t_x = 0$.

Case 3: one-shot game The environment is adapted to a one-shot game. Workers can search and match only once, at the beginning of the game. Workers who do not find a job receives h forever, whereas employed workers received a wage until the job breaks, and h thereafter. $s(e_{ij})m(\theta_j)$ and $q(\theta_j)$ are now transition probabilities instead of hazard rates.

[7] One can also consider a discrete distribution of equally-spaced types on the unit circle.

The definition of submarket tightness (1) and the operator $\mathbb{E}_{i|j}$ are slightly modified to

$$\theta_j = \frac{v_j}{\int_{\mathcal{I}} s(e_{ij})\nu_i di}, \tag{25}$$

$$\mathbb{E}_{i|j} A_{ij} \equiv \frac{\int_{\mathcal{I}} A_{ij} s(e_{ij})\nu_i di}{\int_{\mathcal{I}} s(e_{ij})\nu_i di}. \tag{26}$$

An efficient allocation is now characterised by $\boldsymbol{\theta}$ and $\boldsymbol{e}$ satisfying equations (5) and (6), when substituting ψ_i by h. A decentralised equilibrium is characterized by $\boldsymbol{\theta}$ and $\boldsymbol{e}$ satisfying equations (10) and (13), after operating the same substitution. Proposition 3 remains the same except that h replaces ψ_i. By assuming that workers have the same home production, the surplus differences between workers only comes from productivity differences. Equation (21) writes

$$t^*_{ij} = -\frac{\eta}{1-\eta}(y_{ij} - \mathbb{E}_{i|j} y_{ij}). \tag{27}$$

Accordingly, the optimal tax redistributes from low-productivity to high-productivity jobs within each industry.

2.3 Other policy instruments

A tax system is a convenient policy to examine the gap between the laissez-faire equilibrium and the efficient allocation. I now investigate whether other standard policy instruments amplify or weaken the allocative inefficiency.

Unemployment benefits Consider the same theoretical framework with unemployment benefits b, financed by a lump-sum tax τ. The budget constraint write

$$\tau \int_{\mathcal{I}} (\nu_i - u_i) di = b \int_{\mathcal{I}} u_i di.$$

The introduction of unemployment benefits modifies equation (7). Unemployed workers receive $h_i + b$ instead of h_i. The value of unemployment ψ_i now differs between the efficient allocation and the decentralised equilibrium. With unemployment benefits, ψ_i is increased by $\tilde{b}_i \equiv \frac{b}{1+\int_{\mathcal{J}} \frac{s(e_{ij})m(\theta_j)}{r+\delta_j} dj}$. $\tilde{b}_i < b$ because unemployment benefits reduce the gains from matching $W_{ij} - U_i$. Keeping ψ_i as defined in equation (4), flow workers' surpluses write

$$w_{ij} - \psi_i - \tilde{b}_i = \tilde{\beta}[y_{ij} - t_{ij} - \psi_i - \tau - \tilde{b}_i]. \tag{28}$$

Tax-financed unemployment benefits have ambiguous effects on wages, as described by Pissarides (1985). On the one hand, unemployment benefits improve the bargaining position of workers, driving up wages. On the other hand, the lump-sum tax to finance these benefits pushes down after-tax wages. However, flow workers' surpluses are always decreasing, as both taxes and benefits have the same effect.

Overall, unemployment benefits have ambiguous effects on search efforts. As Decreased match surpluses translate to decreased search efforts. However, search selectivity improves. For instance, large unemployment benefits can turn some matches not profitable anymore, and induce $e_{ij} = 0$ for low-quality jobs.

Subsidising job creation Workers' search strategy impacts job creation through the composition externality. It might be relevant to subsidise job creation to compensate for the profits loss due to mismatch. In addition to the baseline tax scheme, the fiscal authority repays part of the vacancy cost when a job is filled. The cost of a vacancy k_j is thus reduced by the amount a. A lump-sum tax finances this subsidy,

$$\tau \int_{\mathcal{I}} (\nu_i - u_i) di = a \int_{\mathcal{J}} v_j dj.$$

The equivalents of equations (17) and (18) in Lemma 2 are

$$\mathbb{E}_{i|j}\pi_{ij} = (1 - \tilde{\beta}) \left(\mathbb{E}_{i|j}[y_{ij} - t_{ij} - \psi_i - \tau] \right), \tag{29}$$

$$\frac{\mathbb{E}_{i|j} w_{ij}}{r + \delta_j} = \frac{\mathbb{E}_{i|j}[y_{ij} - t_{ij} - \tau]}{r + \delta_j} - \frac{k_j - a}{q(\theta_j)}. \tag{30}$$

The policy has an ambiguous effect on wages. The lump-sum tax reduces industry-mean wages whereas the subsidy component increases them. Relative wages defined by equation (18) are not affected by the policy. Therefore, the policy does not modify the selectivity of job seekers.

Minimum wage At a laissez-faire equilibrium, a minimum wage $\underline{w}$ is equivalent to banning jobs that are not productive enough, when productivity y_{ij} is below $\underline{w}$. A minimum wage can thus be useful when workers search where they should exert no effort at all, namely when $e_{ij} > 0$ in the decentralised equilibrium but $e_{ij} = 0$ in the efficient allocation. The same effect is present with unemployment benefits. The wage setting, however, remains unchanged. A minimum wage does not generate the search disincentives of unemployment benefits. Consequently, this policy is more effective than unemployment benefits in making job seekers more selective.

3 Discussion on model extensions

In this section, I introduce an extension of the model with on-the-job search. I also discuss how the results rely on the assumptions of segmented labour markets and risk-neutral workers.

3.1 On-the-job search

On-the-job search is a relevant margin to analyse job mismatching because it generates wage dispersion in search models (Hornstein et al., 2011). On-the-job search gives incentives for workers to look for less suitable jobs as a stepping stone before finding a better match. Dolado et al. (2009) study this mechanism in a model with undirected random search, but they do not discuss efficiency.

The optimal tax scheme in Proposition 3 extends to an environment with on-the-job search under three hypotheses. I describe the hypotheses and the extended model is solved in appendix C. These hypotheses ensure that on-the-job search do not generate additional inefficiencies. For instance, Gautier et al. (2010) show that on-the-job search can generate a business-stealing externality. A job-to-job transition results in a job destruction with the current employer. Firms do not internalise this job destruction effect when they open vacancies.

First, employed workers face the same submarkets as the unemployed workers. Equation (1) now incorporates the search activities of employed workers in the denominator. The segmentation of the labour market remains unchanged.

Second, a worker cannot break a job contract unless she finds a financial agreement with her current employer. Employed workers thus internalise the cost of job destruction they generate when leaving their current employer. Suppose a worker i is employed in a firm j_0 that generates the present-discounted production $V_{ij_0} \equiv J_{ij_0} + W_{ij_0}$, absent of taxes. The worker receives an offer from another firm of type j_1, producing less $V_{ij_1} < V_{ij_0}$. If the worker's surplus is higher, $W_{ij1} > W_{ij_0}$, a worker could accept this offer even though it is not socially optimal. However, the current employer j_0 would lose the value J_{ij_0}, and thus would block the job-to-job transition. The worker can find an agreement with her current employer by paying at least an amount J_{ij_0}. This amount makes the current employer indifferent between keeping the worker or letting her quit. This device guarantees that the worker can only switch job if the move is socially efficient, $V_{ij_1} > V_{ij_0}$. It also guarantees that on-the-job search efforts maximise the joint value of a match. In particular, search efforts of employed workers do not depend on current wages. Under these hypotheses, employed workers cannot use a job offer to renegociate their wage, such as in models of Postel-Vinay and Robin (2002) and Cahuc et al. (2006). There is no reason for firms to offer efficiency wages as considered by Shimer (2006) and Gautier et al. (2010). Notice the cost of breaching the job contract, J_{ij_0}, can be shared with the poaching firm though Nash bargaining.

Lastly, I assume that there is only one possible wage per match (i, j), independent of the employment history of workers. Workers use the value of unemployment as the outside option when (continuously) bargaining, the Nash-bargaining problem is still $\max(W_{ij} - U_i)^{\beta} J_{ij}^{1-\beta}$. Employed workers have a higher outside option than unemployed workers when meeting a poaching firm. Given that wages are constant, they can extract a hiring bonus $B_{ij_1}^{j_0}$ that depends on the types of her previous and new employer, respectively j_0 and j_1. Notice that workers are risk-neutral and so do not value income smoothing. In addition to taxes on jobs, the government is able to raise $\Theta_{ij_1}^{j_0}$ for each job transition, with the bonus $B_{ij_1}^{j_0}$ being taxed like a wage at a rate τ^w. This design makes sure that bargaining is efficient, and wages are identical for each pair (i, j).

On-the-job search considerably increases the dimensionality of search strategies. Denote $e_{ij_1}^0$ and $e_{ij_1}^{j_0}$ are the search efforts for jobs of type j_1 from, respectively, an unemployed worker of type i and an employed worker of type i currently working in a job j_0. The model accomodates for a different search technology between employed and unemployed workers. I assume $e_{ij_1}^0$ multiplies the baseline job-finding rate by $s(e_{ij_1}^0)$ for unemployed workers, whereas $e_{ij_1}^{j_0}$ multiplies it by $\xi s(e_{ij_1}^{j_0})$. In appendix, I show that the efficient allocation is decentralised with τ^{w*} as defined in (20), and an extension of equation (21). The key interpretation remains: the optimal tax scheme redistributes from low-surplus to high-surplus matches within each industry, pooling together workers irrespective to their previous employment status. Importantly, the match surpluses of unemployed and employed workers differ. The surplus of an unemployed worker is still $V_{ij_1} - U_i = \Omega_{ij_1}$, whereas it is $V_{ij_1} - V_{ij_0} = \Omega_{ij_1} - \Omega_{ij_0}$ for a worker employed in industry j_0. The first-best allocation can still be decentralised despite the increased dimension in the unobserved search efforts.

3.2 Posted wages and labour market segmentation

Nash bargain may appear to be a crucial, and perhaps arbitrary, assumption driving the inefficiency of the decentralised equilibrium. The other popular alternative in search-and-matching models is wage posting. Moen (1997) shows that posted wages decentralise the social optimum, hence the label "competitive search".

I argue that Nash bargain is the most appropriate assumption in an environment where the number of submarkets is *fixed*.[8] Posted wages *per se* are not sufficient to achieve an efficient decentralised equilibrium. The reason is that posted wages is not the only assumption behind the competitive search framework. As Acemoglu and Shimer (1999) explain, competitive search relies also on the ability of workers to identify different submarkets based on the promised wage, and to direct their search accordingly.

Suppose firms post contingent wages. Each firm j commits to a set of wages $\{w_{ij}\}_{i \in \mathcal{I}}$, one wage per worker type. In the competitive search framework, firms are able to open

[8]Another argument against posted wages is that workers have an incentive to renegociate once they are hired. Posted wages require that firms commit and workers are unable to renegociate the wage contract (Moen, 1997).

new submarkets. In particular, it is possible to obtain one active submarket (or more) per pair (i, j).[9] A firm j willing to meet only i-type workers would advertise null wages for workers of type $i' \neq i$ for instance. Perfect segmentation can emerge at equilibrium. In that case, the composition externality disappears because firms know in advance the applicant's type on each submarket. I assume instead that the number of submarkets, one per firm type, is fixed. This restrictions constitutes a constraint to the search-and-matching technology. Even the social planner is constrained by the number of submarkets, and so cannot open a submarket for each pair (i, j). Consequently, the competitive search framework cannot be embedded in this setting.

Suppose wages are posted and the number of submarkets is fixed. A free-rider problem emerges because a j-type firm can post a lower wage than other j-type firms. Doing so, such a firm makes higher profits while finding a worker at the same rate. Submarkets are defined by firm's type in my setting, whereas they are defined by wages in the competitive search framework. In the competitive search framework, a firm cannot free-ride because it would leave the submarket for a new one by posting a different wage. When firms can free-ride, the equilibrium strategy is to post the lowest acceptable wages, $w_{ij} = \psi_i$. This equilibrium is equivalent to the Nash-bargain equilibrium with $\beta = 0$. Interestingly, coordination among firms cannot help to eradicate the inefficiency. If same-type firms can coordinate, they would extract a monopsony rent. In the competitive search framework, such a rent is prevented by the threat of a new submarket being opened. Posted wages are therefore not relevant when the number of submarkets is exogenous.

The crucial hypothesis is the exogenous segmentation of the labour market rather than Nash-bargained wages. It is worth noting that the composition externality does not arise in the two extreme case when the labour market is fully segmented or not segmented. In the first case, there is a submarket for each pair (i, j). In the second case, all types meet with one meeting function. In both cases, the decentralised equilibrium with Nash-bargained wages is efficient under the Hosios-Pissarides condition.

3.3 Risk aversion

Risk aversion introduces two new motives for the social planner. First, the randomness of the search process creates a wage risk. More dispersed wages make workers worse-off. Second, workers suffer from the income loss when unemployed, the unemployment risk. There is thus a need for both redistribution as for Golosov et al. (2013), and unemployment insurance as for Landais et al. (2018). In that case, the planner cares about the pay-off profile contrary to the risk-neutrality benchmark. The first-best allocation with risk aversion is characterised by the same steady-state allocation $(\boldsymbol{\psi}, \boldsymbol{\theta}, \boldsymbol{e})$ as in Definition 2,

[9] Decreuse and Zylberberg (2011) consider the same search technology with decreasing marginal returns with homogeneous agents in a competitive search environment. There is a continuum of submarkets opened at equilibrium.

and pay-offs that equalise flow utilities across individuals.

The objective of redistribution therefore goes against the objective of efficiency that requires a steep wage profile to incentivise search efforts. To maximise efficiency, the social planner would like the wage profile with a slope equal to 1 with production as in condition (15). To maximise equity, it would like a constant wage profile. Consequently, there is no equivalent of Proposition 1 with risk-averse workers. If the social planner wants to maximise welfare with a control on wages, it can only reach a second-best allocation. Determining the formula of the second-best pay-off profile would bring us beyond the scope of the paper. This problem is left for future research.

4 Numerical analysis

The numerical analysis has two purposes. First, it assesses the magnitude of the output loss of the laissez-faire decentralised equilibrium compared to the social optimum. Second, it investigates the features of the optimal tax scheme.

The model with on-the-job search is calibrated on occupational mismatch in the United States. I follow Sahin et al. (2014) and consider the 2-digit level occupations defined by the Standard Occupational Classification System. Each occupation corresponds to a sector $j \in \mathcal{J} = \{1, ..., 22\}$. Heterogeneity among workers consists in occupational comparative advantages. Job productivity can only takes two values for each occupation depending on the worker's type: $y_j^H > y_j^L > h$. Each worker $i \in \mathcal{I} = \{1, ..., 22\}$ is able to produce at the high level of productivity in only one sector, for which $j = i$. Home production $h_i = h$ is identical across workers. The production function writes

$$y_{ij} = y_j^H \mathbf{1}_{j=i} + y_j^L \mathbf{1}_{j \neq i}.$$

Uren (2006) suggests that comparative advantages eliminate the risk of multiplicity. I do not prove formally that this setting always lead to a unique equilibrium, but I do not find multiple equilibria numerically.[10]

The bargaining power β and the matching elasticity η are critical parameters determining the extent of output loss of the equilibrium compared to the social optimum. Given the absence of consensus on their empirical values, the model is calibrated for nine different values of (η, β). The same calibration strategy is applied to each pair. The annual interest rate is fixed at 5%. The meeting function writes $m(\theta) = m_1 \theta^\eta$. The search function is supposed isoleastic, $s(e) = s_1 e^\epsilon$. Multiplying s_1 by 2 is equivalent to divide m_1 and k_j by 2. The search multiplier s_1 can thus be normalised to 1. Multiplying m_1 by a coefficient is equivalent to dividing k_j and to moving the equilibrium market tightnesses

[10] Decreuse (2008) also discusses the possibility of multiple equilibria. He provides a sufficient condition for uniqueness with horizontal heterogeneity.

θ_j. As I do not use any information about the measure of vacancies nor market tightnesses, m_1 can be normalised to 1. k_j thus captures sector-specific search frictions. The total measure of workers is normalised to 1. I suppose employed and unemployed workers have the same search technology, $\xi = 1$, as Christensen et al. (2005) do.

The remaining parameters are calibrated using the Current Population Survey data from January 2017 to December 2018. The data are supposed to correspond to the decentralised equilibrium without taxes. I extract information about the distribution of employed workers and average wages by occupation, and monthly transitions between occupations and unemployment. Home production h is fixed at 40% of the average wage, which is the replacement rate suggested by Shimer (2005). A worker's type cannot be directly observed in the data. I approximate the distribution of workers over types ν_i by the empirical distribution of employed workers in each occupation. The job destruction rates δ_j are defined by the monthly employment-to-unemployment transition rates by occupation, averaged over the period 2017-2018. The productivity levels y_j^H and y_j^L, the cost of vacancies k_j and the search elasticity ϵ are obtained by minimising the distance between well-chosen moments in the data and their theoretical counterpart. There are 67 parameters to determine.[11]

There are three types of moments: occupation-mean wages (22 moments), monthly unemployment-to-employment transition rates (22x22 moments) and monthly employment-to-employment transition rates (22x21 moments). The moments are averaged over the 2-year period. For each unemployed worker, I use information about the previous occupation held. This explains why there are 22x22 moments in the second series of moments. Employment-to-employment transitions are conditional on changing occupation. Equivalently, job-to-job transition within the same occupation are ignored because the model is not suited to explain these transitions. This explains why there are 22x21 moments in the third series of moments.

I use a weighted sum of square distances to build the criterion to minimise. Weights are defined as follows. For each occupation-mean wage, the distance is defined as the difference between the theoretical mean wage and its empirical counterpart, divided by the mean wage observed among the 22 occupations. I proceed the same way to compute distances between transition rates. Each distance is then weighted by the number of moments per type of moments in order to equalise the contribution of each type to the criterion.[12] Denote $\{m_{1j}\}$, $\{m_{2j_0j_1}\}$ and $\{m_{3j_0j_1}\}$ the three sets of moments, using the superscript *theo* and *emp* for theoretical and empirical moments respectively. Define $\bar{m}_1$,

[11] Due to limited computational power, I do not incorporate ν_i and δ_j to the distance minimisation algorithm although it would be more statistically efficient.

[12] We cannot divide each distance by the empirical moment because some of them are equal to zero. I therefore divide by the mean empirical moment.

$\bar{m}_2$ and $\bar{m}_3$ the mean for each set of moments. The criterion to minimise writes

$$\frac{1}{22}\sum_{j=1}^{22}\left(\frac{m_{1j}^{theo}-m_{1j}^{emp}}{\bar{m}_1^{emp}}\right)^2+\frac{1}{22\times 22}\sum_{j_0=1}^{22}\sum_{j_1=1}^{22}\left(\frac{m_{2j_0j_1}^{theo}-m_{2j_0j_1}^{emp}}{\bar{m}_2^{emp}}\right)^2 \\ +\frac{1}{22\times 21}\sum_{j_0=1}^{22}\sum_{\substack{j_1=1\\ j_1\neq j_0}}^{22}\left(\frac{m_{3j_0j_1}^{theo}-m_{3j_0j_1}^{emp}}{\bar{m}_3^{emp}}\right)^2.$$

In the model, the type of an unemployed worker can differ from the type of her previous occupation. I carefully take this aspect into account when defining the theoretical moments. In particular, define u_{ij_0} the measure of unemployed workers of type i previously employed in occupation j_0. The worker's type i is not observable but j_0 is. At a steady state,

$$u_{ij_0}=\frac{\delta_{j_0}n_{ij_0}}{\int_{\mathcal{J}}s(e^0_{ij_1})m_{j_1}dj_1}.$$

Theoretical moments are given in Table 1. I provide an intuition on the way moments identify the model parameters. If mismatched workers were not productive enough to generate a positive surplus, for instance $y_j^L=0$, then workers would only apply to their best jobs $j=i$. There would be no mismatch, no employment-to-employment transitions, and $u_{ij}=0$ for $j\neq i$ at equilibrium. The first two series of moments would identify productivity levels y_j^H and vacancy costs k_j. The presence of job-to-job transitions can only be explained by larger values of y_j^L in the model. Therefore, the third series of moments bring identifying information about y_j^L. Lastly, the search cost elasticity ϵ is identified by variations in job-finding rates related to variations in the gains from obtaining a new job. Observing enough transitions towards employment therefore identifies ϵ.

Empirical moment	Theoretical counterpart
Average wage in j	$\frac{\int_{\mathcal{J}}w_{ij}n_{ij}dj}{\int_{\mathcal{J}}n_{ij}dj}$
Transition rate from U_{j_0} to E_{j_1}	$\frac{\int_{\mathcal{I}}s(e^0_{ij_1})m(\theta_{j_1})u_{ij_0}di}{\int_{\mathcal{I}}u_{ij_0}di}$
Transition rate from E_{j_0} to E_{j_1}	$\frac{\int_{\mathcal{I}}\xi s(e^{j_0}_{ij_1})m(\theta_{j_1})n_{ij_0}di}{\int_{\mathcal{I}}n_{ij_0}di}$

Table 1: Moments to determine y_{ij}, k_j and ϵ

Note. E_j means employment in occupation j, and U_j means unemployment of workers previously employed in occupation j.

The results for the nine calibrated models are provided in Table 2. The nine models perform equally well on matching the moments. The moments fitting strategy thus does not provide insights for choosing particular values for β and η. I report three statistics on

	Models for fixed matching elasticity and bargaining power								
	(A)	(B)	(C)	(D)	(E)	(F)	(G)	(H)	(I)
Fixed parameters									
matching elasticity η	0.25	0.25	0.25	0.5	0.5	0.5	0.75	0.75	0.75
bargaining power β	0.25	0.5	0.75	0.25	0.5	0.75	0.25	0.5	0.75
Distance to moments									
occupation-mean wages	0.003	0.003	0.003	0.004	0.018	0.017	0.003	0.002	0.005
U to E rates	0.3	0.3	0.31	0.29	0.3	0.3	0.29	0.3	0.31
E to E rates	1.49	1.48	1.47	1.41	1.37	1.4	1.45	1.47	1.4
Calibrated parameters									
mean perfect-match productivity	24.7	21.4	20.5	24.5	21.2	20.3	25.5	21.5	20.3
mean mismatch penalty (%)	59	55	55	57	54	53	57	57	55
search elasticity ϵ	0.45	0.43	0.4	0.47	0.45	0.42	0.49	0.39	0.39
Efficiency loss (%)	15	7.9	1.8	14.6	8	4.9	15.1	8.1	5.2
Unemployment rate (%)									
equilibrium	2.1	2.1	2.1	2.1	2.1	2.1	2.1	2.1	2.1
optimum	0.5	0.6	1.9	0.7	0.4	0.5	0.4	0.2	0.3
Share of mismatches (%)									
equilibrium	11	11	11	13	15	12	19	11	18
optimum	4	1	10	4	11	9	16	6	16
Mean-min ratio									
equilibrium	2.51	2.51	2.55	2.53	2.47	2.51	2.51	2.57	2.54
optimum	3.11	2.75	2.61	3.1	2.69	2.6	3.11	2.8	2.64
Productivity-taxes correlation	-0.85	-0.72	-0.82	-0.79	-0.75	-0.74	-0.68	-0.77	-0.65

Table 2: Simulated outcomes for different matching elasticities, bargaining parameters and slopes of the production function

Note. A model is calibrated for each tuple (η, β) at the decentralised equilibrium. The average square distance between moments is reported by type of moments, the three statistics sum up to the criterion to minimise. The remaining statistics are obtained with the calibrated parameters, either at the laissez-faire equilibrium, or at the social optimum.

the calibrated parameters. As the bargaining power increases, the mean perfect-match productivity $\frac{1}{22}\sum_{j=1}^{22} y_j^H$ decreases so that wages match the same values. The average penalty from being mismatched, $\frac{1}{22}\sum_{j=1}^{22} \frac{y_j^H - y_j^L}{y_j^H}$, is between 50% and 60%. The search elasticity is calibrated between 0.4 and 0.5.

Before comparing the gaps between the decentralised equilibrium and the efficient allocation, I discuss the values of unemployment rate and statistics of mismatch for the calibrated model, at equilibrium. The calibrated models predict an equilibrium unemployment rate at 2.1%. It is lower than the unemployment rate provided by the Bureau of Labor Statistics for this period. In the model, the unemployment rate is obtained from flows of workers and the hypothesis of steady state. Mechanically, the unemployment rate is low because job-finding rates are high and job separation rates are low in the 2-year period considered. The share of mismatched workers, for which $j \neq i$, corresponds to $\frac{\sum_{i\neq j} n_{ij}}{\sum_{i,j} n_{ij}}$. Between 10 and 20% of workers are not employed in their best occupation. I also consider the mean-min ratio proposed by Hornstein et al. (2011) to describe wage dispersion due to search frictions. The mean-min ratio is averaged per type of worker, $\frac{\sum_{i,j} n_{ij}\mathrm{Mm}_i}{\sum_{i,j} n_{ij}}$ with $Mm_i = \frac{\sum_j n_{ij} w_{ij}}{\min_j\{w_{ij}\}.\sum_j n_{ij}}$. These ratios are large in all the models, larger than the empirical ratios comprised between 1.7 to 2. These large ratios are explained by the strong mismatch penalty faced by workers. Intuitively, if workers are more than twice productive in their perfect-match occupation, mean-min ratios could exceed 2.

The efficiency loss is the relative distance between net output produced at the social optimum and at the laissez-faire equilibrium. The efficiency loss ranges from 2% to 15%. It increases with the matching elasticity, and decreases with the bargaining power. These patterns are consistent with the comment of Proposition 2: the laissez-faire allocation is asymptotically efficient when η tends to 0 and β to 1. The model (C) exhibiting 2% of efficiency loss is the closest to this asymptotic case. The efficiency loss diminishes as β increases. For a given matching elasticity η, the Hosios-Pissarides condition does not minimise efficiency, for instance in columns (E) and (G). The welfare loss is therefore mainly driven by the composition externality compared to the standard search externalities.

The optimal policy significantly reduces unemployment. This is not a general result from the theory. On the one hand, the optimal policy makes workers more selective, and so unemployment durations could increase. On the other hand, firms could open more vacancies once the composition externality vanishes. Here, unemployment reduces because the response of firms dominates: they create so many new vacancies that the job-finding rate increases overall.

As expected, there are many less mismatches at the social optimum. The mean-min ratio, however, is larger than at the laissez-faire equilibrium. This trend reflects a change in the distribution of workers per jobs. At the social optimum, workers are more selective leading to an increase in the mean wages per type of worker. The minimum wage is barely affected by a change in search strategies if workers still look for low-productive matches.

The degree of anti-redistribution involved by the optimal tax scheme is captured by the correlation coefficient between job productivities y_{ij} and taxes t^*_{ij}, weighted by the distribution of workers:

$$\frac{\sum_{i,j} n_{ij}(y_{ij} - \bar{y})(t^*_{ij} - \bar{t})}{\sqrt{\sum_{i,j} n_{ij}(y_{ij} - \bar{y})^2}\sqrt{\sum_{i,j} n_{ij}(t^*_{ij} - \bar{t})^2}}.$$

$\bar{y}$ and $\bar{t}$ denote the average productivity levels and taxes weighted by the distribution of workers. Productivity levels and taxes are strongly negatively correlated, with a coefficient between -0.65 and -0.85. This coefficient suggests that the tax scheme has a significant anti-redistributive effect.

5 Conclusion

In general, optimal redistribution policies balance the gains from equity and insurance on the one hand, and the efficiency cost of economic distortions on the other hand. Equilibrium inefficiencies requiring Pigouvian intervention are mainly ignored. This article examines the Pigouvian motive of taxation in the presence of inefficient job search activities. Here, the Pigouvian motive goes against the equity motive for redistribution. I show this result by isolating the Pigouvian motive from the equity motive. Without social preferences for redistribution, the optimal tax scheme is anti-redistributive in many cases, including the models of the numerical analysis. By ignoring the inherent inefficiency of search activities, the policy maker therefore understates the efficiency cost of redistribution. In practice, the magnitude of the inherent inefficiency depends on two key channels: i) the response of job creation to expected job productivity, ii) the productivity loss from mismatching.

By abstracting from the equity motive of taxation, the optimal tax scheme is not recommended to policy makers. Two avenues can be considered to study the practical implementation of the optimal tax scheme. First, social preferences for equity should be introduced in the model. Second, one could relax the assumption on observable types by the fiscal authority. In the model, the authority does not observe efforts but can tax jobs depending on their type. The traditional approach in optimal taxation would be to consider non-linear taxation on wages, without the ability of the government to observe the types of agent.

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A Proof of Lemma 1

Let $H(\boldsymbol{\theta}, \boldsymbol{e}, \boldsymbol{n}, \boldsymbol{u}, \boldsymbol{\lambda}, \boldsymbol{\mu})$ be the current-value Hamiltonian of the planner's program, with λ_{ij} and μ_i the multipliers of the constraints on n_{ij} and u_i. Hereafter, we denote it H for ease of reading. The control equations of the dynamic program are

$$\dot{n}_{ij} = s(e_{ij})m(\theta_j)u_i - \delta_j n_{ij}, \qquad \dot{u}_i = \int_{\mathcal{J}} \delta_j n_{ij} dj - \int_{\mathcal{J}} s(e_{ij})m(\theta_j)u_i dj. \tag{31}$$

The Hamiltonian is defined as

$$\begin{aligned} H = & \int_{\mathcal{I}\times\mathcal{J}} y_{ij} n_{ij} didj + \int_{\mathcal{I}} h_i u_i di - \int_{\mathcal{I}\times\mathcal{J}} k_j \theta_j s(e_{ij}) u_i didj - \int_{\mathcal{I}\times\mathcal{J}} e_{ij} u_i didj \\ & + \int_{\mathcal{I}\times\mathcal{J}} \lambda_{ij} \left[s(e_{ij})m(\theta_j)u_i - \delta_j n_{ij} \right] didj \\ & + \int_{\mathcal{I}} \psi_i \left[\int_{\mathcal{J}} \delta_j n_{ij} dj - \left(\int_{\mathcal{J}} s(e_{ij})m(\theta_j) dj \right) u_i \right] di. \end{aligned}$$

The two costate equations at steady state write $\frac{\partial H}{\partial n_{ij}} = r\lambda_{ij}$ and $\frac{\partial H}{\partial u_i} = r\mu_i$. They write

$$r\lambda_{ij} = y_{ij} - \delta_j(\lambda_{ij} - \mu_i), \tag{32}$$

$$r\mu_i = h_i + \int_{\mathcal{J}} s(e_{ij})m(\theta_j)\left[\lambda_{ij} - \mu_i\right] dj - \int_{\mathcal{J}} k_j s(e_{ij})\theta_j dj - \int_{\mathcal{J}} e_{ij} dj. \tag{33}$$

The flow value of an unemployed worker is by defintion $\psi_i = r\mu_i$. Equation (4) derives from the second costate equation, after having substituted λ_{ij} by its expression.

The optimal market tightnesses, $\boldsymbol{\theta}$, and search strategies, $\boldsymbol{e}$, maximise the Hamitonian H. The Hamiltonian is linear with the state variables: $H = \int_{\mathcal{I}\times\mathcal{J}} \frac{\partial H}{\partial n_{ij}} n_{ij} didj + \int_{\mathcal{I}} \frac{\partial H}{\partial u_i} u_i di$. The first term does not depend on $\boldsymbol{\theta}$ nor on $\boldsymbol{e}$. The planner's problem is thus equivalent to maximising $\int_{\mathcal{I}} \frac{\partial H}{\partial u_i} u_i di$. $\Psi_i(\boldsymbol{\theta}, \boldsymbol{e}, \psi_i) \equiv \frac{\partial H}{\partial u_i}$ in Lemma 1.

B General wage taxation

Consider a general differentiable taxation function $t_{ij}(w)$. This taxation potentially generates different worker's shares of the surplus. Define $\tilde{\beta}_{ij}(w) = \frac{\beta}{\beta+(1-\beta)(1+t'_{ij}(w))}$.

Lemma 2 can be generalised with the three following equations:

$$\mathbb{E}_{i|j}\pi_{ij} = \mathbb{E}_{i|j}[(1-\tilde{\beta}_{ij}(w_{ij}))(y_{ij}-t_{ij}(w_{ij})-\psi_i)], \tag{34}$$

$$\frac{\mathbb{E}_{i|j}w_{ij}}{r+\delta_j} = \frac{\mathbb{E}_{i|j}[y_{ij}-t_{ij}(w_{ij})]}{r+\delta_j} - \frac{k_j}{q(\theta_j)}, \tag{35}$$

$$\begin{aligned} w_{ij} - \mathbb{E}_{i|j}w_{ij} = {} & \tilde{\beta}_{ij}(w_{ij})(y_{ij}-t_{ij}(w_{ij})) + (1-\tilde{\beta}_{ij}(w_{ij}))\psi_i \\ & - \mathbb{E}_{i|j}[\tilde{\beta}_{ij}(w_{ij})(y_{ij}-t_{ij}(w_{ij})) + (1-\tilde{\beta}_{ij}(w_{ij}))\psi_i]. \end{aligned} \tag{36}$$

To satisfy condition (15), it must be first that $\mathbb{E}_{i|j}t_{ij}(w_{ij}) = 0$. Second, the last equation above can be written

$$\tilde{\beta}_{ij}(w_{ij})t_{ij}(w_{ij}) = -(1-\beta_{ij})(y_{ij}-\psi_i) + \mathbb{E}_{i|j}[(1-\beta_{ij})(y_{ij}-t_{ij}(w_{ij})-\psi_i)]. \tag{37}$$

Condition (14) requires

$$\eta\mathbb{E}_{i|j}[y_{ij}-\psi_i] = \mathbb{E}_{i|j}[(1-\beta_{ij})(y_{ij}-t_{ij}(w_{ij})-\psi_i)]. \tag{38}$$

Assuming that $\tilde{\beta}_{ij}(w_{ij})$, we obtain

$$\mathbb{E}_{i|j}t_{ij}(w_{ij}) = \mathbb{E}_{i|j}\left[\frac{\eta-(1-\beta_{ij})}{\beta_{ij}}(y_{ij}-\psi_i)\right], \tag{39}$$

which must be equal to 0. As long as $y_{ij}-\psi_i > 0$ within the expectation, necessarily $\tilde{\beta}_{ij}(w_{ij}) = 1-\eta$. Thus the wage should be taxed with a fixed rate. We obtain the same solution as in Proposition 3.

C Extension with on-the-job search

An unemployed worker of type i searches for a job of type j_1 with effort $e^0_{ij_1}$. An employed worker of type i employed in a job of type j_0 searches for a job of type j_1 with effort $e^{j_0}_{ij_1}$. In the subsequent analysis, I will use j_0 to denote the previous employer's type, and j_1 the new one. In presence of on-the-job search, market tightness for each industry j_1 is defined by

$$\theta_j = \frac{v_j}{\int_{\mathcal{I}} s(e^0_{ij})u_i di + \int_{\mathcal{I}\times\mathcal{J}} \xi s(e^{j_0}_{ij})n_{ij_0} di dj_0}. \tag{40}$$

Stocks of jobs (i,j) obey new dynamics to account for the inflows and outflows from job-to-job transitions,

$$\dot{n}_{ij} = s(e_{ij}^0)m(\theta_j)u_i + \int_{\mathcal{J}} \xi s(e_{ij}^{j_0})m(\theta_j)n_{ij_0}dj_0 - \delta_j n_{ij} - \int_{\mathcal{J}} \xi s(e_{ij_1}^{j})m(\theta_{j_1})n_{ij}dj_1. \quad (41)$$

Efficient allocation The current-value welfare Y_t writes

$$\int_{\mathcal{I}\times\mathcal{J}} y_{ij}n_{ij}didj + \int_{\mathcal{I}} h_i u_i di - \int_{\mathcal{J}} k_j v_j dj - \int_{\mathcal{I}\times\mathcal{J}} e_{ij}^0 u_i didj - \int_{\mathcal{I}\times\mathcal{J}\times\mathcal{J}} e_{ij}^{j_0} n_{ij_0} didj_0 dj.$$

With the same notations as in appendix A, the current-value Hamiltonian of the planner's program is defined as

$$\begin{aligned}
H = & \int_{\mathcal{I}\times\mathcal{J}} y_{ij}n_{ij}didj + \int_{\mathcal{I}} h_i u_i di - \int_{\mathcal{I}\times\mathcal{J}} k_j\theta_j s(e_{ij}^0)u_i didj - \int_{\mathcal{I}\times\mathcal{J}\times\mathcal{J}} k_j\theta_j\xi s(e_{ij}^{j_0})n_{ij_0}didj_0dj \\
& - \int_{\mathcal{I}\times\mathcal{J}} e_{ij}^0 u_i didj - \int_{\mathcal{I}\times\mathcal{J}\times\mathcal{J}} e_{ij}^{j_0} n_{ij_0} didj_0 dj \\
& + \int_{\mathcal{I}\times\mathcal{J}} \lambda_{ij}\left[s(e_{ij}^0)m(\theta_j)u_i + \int_{\mathcal{J}} \xi s(e_{ij}^{j_0})m(\theta_j)n_{ij_0}dj_0 - \delta_j n_{ij} - \int_{\mathcal{J}} \xi s(e_{ij_1}^{j})m(\theta_{j_1})n_{ij}dj_1\right] didj \\
& + \int_{\mathcal{I}} \mu_i\left[\int_{\mathcal{J}} \delta_j n_{ij}dj - \left(\int_{\mathcal{J}} s(e_{ij}^0)m(\theta_j)dj\right)u_i\right] di.
\end{aligned}$$

The two costate equations at the steady state write $\frac{\partial H}{\partial n_{ij_0}} = r\lambda_{ij_0}$ and $\frac{\partial H}{\partial u_i} = r\mu_i$. It follows

$$r\lambda_{ij} = y_{ij} + \int_{\mathcal{J}} \xi s(e_{ij_1}^{j})m(\theta_{j_1})[\lambda_{ij_1} - \lambda_{ij}]dj_1 - \int_{\mathcal{J}} k_{j_1}\theta_{j_1}\xi s(e_{ij_1}^{j})dj_1 - \delta_j(\lambda_{ij} - \mu_i) - \int_{\mathcal{J}} e_{ij_1}^{j} dj_1, \quad (42)$$

$$r\mu_i = h_i + \int_{\mathcal{J}} s(e_{ij}^0)m(\theta_j)[\lambda_{ij} - \mu_i]dj - \int_{\mathcal{J}} k_j\theta_j s(e_{ij}^0)dj - \int_{\mathcal{J}} e_{ij}^0 dj. \quad (43)$$

Optimal market tightnesses, $\boldsymbol{\theta}$, and search strategies, $\boldsymbol{e}$, maximize the Hamitonian H. The Hamiltonian is linear with the state variables: $H = \int_{\mathcal{I}\times\mathcal{J}} \frac{\partial H}{\partial n_{ij}} n_{ij}didj + \int_{\mathcal{I}} \frac{\partial H}{\partial u_i} u_i di$. With on-the-job search, we do not have an equivalent of Lemma 1 because $\frac{\partial H}{\partial n_{ij}}$ depends on endogenous variables. Optimal market tightness here solves

$$\frac{k_j}{q(\theta_j)} = \eta \frac{\int_{\mathcal{I}\times\mathcal{J}} \xi s(e_{ij}^{j_0})(\lambda_{ij} - \lambda_{ij_0})n_{ij_0}didj_0 + \int_{\mathcal{I}} s(e_{ij}^0)(\lambda_{ij} - \mu_i)u_i di}{\int_{\mathcal{I}\times\mathcal{J}} \xi s(e_{ij}^{j_0})n_{ij_0}didj_0 + \int_{\mathcal{I}} s(e_{ij}^0)u_i di}, \quad (44)$$

or $\theta_j = 0$.

The fraction in the right-hand side is the expected match surplus within industry j. This equation is the equivalent of (5). The optimal search efforts are given by the equivalent of equation (6):

$$\xi s'(e_{ij}^{j_0})m(\theta_j)\left(\lambda_{ij}-\lambda_{ij_0}-\frac{k_j}{q(\theta_j)}\right)=1, \qquad \text{or } e_{ij}^{j_0}=0, \tag{45}$$

$$s'(e_{ij}^{0})m(\theta_j)\left(\lambda_{ij}-\mu_i-\frac{k_j}{q(\theta_j)}\right)=1, \qquad \text{or } e_{ij}^{0}=0. \tag{46}$$

The terms within the parentheses are ex-ante match surpluses following a transition.

Definition 2 has the following equivalent. A steady-state allocation is efficient if values $\boldsymbol{\lambda}$ and $\boldsymbol{\mu}$ satisfy (42) and (43), market tightnesses $\boldsymbol{\theta}$ satisfy (44) and search efforts $\boldsymbol{e}$ satisfy (45) and (46).

Decentralised equilibrium The fiscal authority raises taxes t_{ij} for each job as in the benchmark model without on-the-job search. $B_{ij_1}^{j_0}$ is the endogenous (after-tax) bonus bargained between the new employer and the job switcher. The fiscal authority also taxes $\Theta_{ij_1}^{j_0}=\tau^w B_{ij_1}^{j_0}+\Theta_{ij_1}^{f,j_0}$ for each job-to-job transition from j_0 to j_1. $\Theta_{ij_1}^{f,j_0}$ is a lump-sum tax, and τ^w the proportional tax on labour incomes.

The asset values are defined by the following Bellman equations:

$$rU_i = h_i + \int_{\mathcal{J}} s(e_{ij}^0)m(\theta_j)\left[W_{ij}-U_i\right]dj - \int_{\mathcal{J}} e_{ij}^0 dj, \tag{47}$$

$$rW_{ij} = w_{ij} + \int_{\mathcal{J}} \xi s(e_{ij_1}^{j})m(\theta_{j_1})[W_{ij_1}+B_{ij_1}^{j}-W_{ij}-J_{ij}]dj_1 + \delta_j\left(U_i-W_{ij}\right) - \int_{\mathcal{J}} e_{ij_1}^{j} dj_1, \tag{48}$$

$$rJ_{ij} = \pi_{ij} - \delta_j J_{ij} + \int_{\mathcal{J}} \xi s(e_{ij_1}^{j})m(\theta_{j_1})[J_{ij}-J_{ij}]dj_1. \tag{49}$$

W_{ij} and J_{ij} are the values from matching net of the bonus payment. Following a job-to-job transition from j_0 to j_1, the worker obtains $W_{ij_1}+B_{ij_1}^{j_0}$, renounces to W_{ij_0}, and pays J_{ij_0} to its previous employer. The new employer receives expected profits net of the bonus and taxes $J_{ij_1}-B_{ij_1}^{j_0}-\Theta_{ij_1}^{j_0}$. The previous employer loses J_{ij_0} from job destruction but receives the payment J_{ij_0} in compensation.

Workers choose their search efforts according to

$$\xi s'(e_{ij_1}^{j_0})m(\theta_{j_1})[W_{ij_1}+B_{ij_1}^{j_0}-W_{ij_0}-J_{ij_0}]=1, \qquad \text{or } e_{ij_1}^{j_0}=0, \tag{50}$$

$$s'(e_{ij}^{0})m(\theta_j)\left(W_{ij}-U_i\right)=1, \qquad \text{or } e_{ij}^{0}=0. \tag{51}$$

Define the operator $\mathbb{E}_{ip|j}$ as

$$\mathbb{E}_{ip|j}A_{ij}^{p} = \frac{\int_{\mathcal{I}\times\mathcal{J}} \xi s(e_{ij_1}^{j_0})A_{ij_1}^{j_0} n_{ij_0} di dj_0 + \int_{\mathcal{I}} s(e_{ij_1}^{0})A_{ij_1}^{0} u_i di}{\int_{\mathcal{I}\times\mathcal{J}} \xi s(e_{ij_1}^{j_0}) n_{ij_0} di dj_0 + \int_{\mathcal{I}} s(e_{ij_1}^{0}) u_i di}.$$

With conventions $B^0_{ij} = \Theta^0_{ij} = 0$, the no-arbitrage condition for job creation writes

$$\frac{k_j}{q(\theta_j)} = \mathbb{E}_{ip|j}[J_{ij} - B^p_{ij} - \Theta^p_{ij}]. \tag{52}$$

Denote the value of taxes raised from each job $T_{ij} \equiv \frac{t_{ij}}{r+\delta_j} = \frac{y_{ij}-w_{ij}-\pi_{ij}}{r+\delta_j}$. The value of a match writes $V_{ij} = W_{ij} + J_{ij} + T_{ij}$. The Bellman equations give

$$rV_{ij} = y_{ij} + \int_{\mathcal{J}} \xi s(e^j_{ij_1}) m(\theta_{j_1})[W_{ij_1} + B^j_{ij_1} + T_{ij} - V_{ij}]dj_1 + \delta_j (U_i - V_{ij}) - \int_{\mathcal{J}} e^j_{ij_1} dj_1. \tag{53}$$

The values U_i and V_{ij} are directly comparable to the multipliers μ_i and λ_{ij}. Define the match surplus Ω^p_{ij} depending on the previous employee's situation p:

$$\Omega^0_{ij} = V_{ij} - U_i,$$
$$\Omega^{j_0}_{ij_1} = V_{ij_1} - V_{ij_0}$$

The decentralised equilibrium is efficient if the three following conditions are fulfilled:

1. $\mathbb{E}_{ip|j}(J_{ij} - B^p_{ij} - \Theta^p_{ij}) = \eta \mathbb{E}_{ip|j} \Omega^p_{ij}$,
2. $W_{ij} - U_i = \Omega^0_{ij} - \frac{k_j}{q(\theta_j)}$,
3. $W_{ij_1} + B^{j_0}_{ij_1} + T_{ij_0} - V_{ij_0} = \Omega^{j_0}_{ij_1} - \frac{k_{j_1}}{q(\theta_{j_1})}$

The two first conditions are the extensions of Proposition 1 written in terms of surplus. The last condition ensures that the surplus from a job-to-job transition is efficiently valued by private agents.

I now turn to Nash bargaining. The bargaining of the wage is similar to the bargaining in the benchmark model,

$$J_{ij} = (1-\tilde{\beta})(\Omega^0_{ij} - T_{ij}) \qquad \text{and} \qquad W_{ij} - U_i = \tilde{\beta}(\Omega^0_{ij} - T_{ij}). \tag{54}$$

The bargaining of the bonus provides a new equation,

$$\begin{aligned} J_{ij_1} - B^{j_0}_{ij_1} - \Theta^{j_0}_{ij_1} &= (1-\tilde{\beta})(\Omega^{j_0}_{ij_1} - [\Theta^{j_0}_{ij_1} + T_{ij_1} - T_{ij_0}]), \text{ and} \\ W_{ij_1} + B^{j_0}_{ij_1} + T_{ij_0} - V_{ij_0} &= \tilde{\beta}(\Omega^{j_0}_{ij_1} - [\Theta^{j_0}_{ij_1} + T_{ij_1} - T_{ij_0}]). \end{aligned} \tag{55}$$

$\Theta^{j_0}_{ij_1} + T_{ij_1} - T_{ij_0}$ is the capital gain the fiscal authority makes when a worker switches job. The bargained bonus writes $B^{j_0}_{ij_1} = (1-\tilde{\beta})(\Omega^0_{ij_0} - T_{ij_0}) - \tilde{\beta}\Theta^{j_0}_{ij_1}$. Notice the bonus exactly compensates the cost of breaching the contact J_{ij_0} at the laissez-faire equilibrium.

Optimal taxation The optimal wage tax rate τ_w^* is defined by condition (20) to obtain $1 - \tilde{\beta} = \eta$. The baseline taxation T_{ij} and the taxation on job transitions $\Theta_{ij_1}^{j_0}$ satisfy:

$$T_{ij} = -\frac{\eta}{1-\eta}(\Omega_{ij}^0 - \mathbb{E}_{ip|j}\Omega_{ij}^p), \tag{56}$$

$$\Theta_{ij_1}^{j_0} + T_{ij_1} - T_{ij_0} = -\frac{\eta}{1-\eta}(\Omega_{ij_1}^{j_0} - \mathbb{E}_{ip|j_1}\Omega_{ij_1}^p). \tag{57}$$

This implies $\Theta_{ij_1}^{j_0} = \frac{\eta}{1-\eta}\mathbb{E}_{ip|j_0}\Omega_{ij_0}^p$, which only depends on j_0. There are two departures from the optimal taxation in the benchmark model. First, the fiscal authority accounts for match surpluses following job-to-job transitions in the expected match surplus $\mathbb{E}_{ip|j}\Omega_{ij}^p$. Second, each job-to-job transition is taxed. Overall, the fiscal authority clears its budget constraint.