Optimal Taxation to Correct Job Mismatching*

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Abstract

This paper presents a new efficiency argument for an accommodating taxation policy on high incomes. Job seekers, applying to different segments of a frictional labor market, do not internalize the consequences of mismatch on the entry decision of firms. Workers are not selective enough, resulting in a lower average job productivity and suboptimal job creation. When workers have symmetric comparative advantages, the first-best output-maximizing allocation can be decentralized through a simple anti-redistributive taxation. An income tax guarantees an optimal sharing of the match surplus and a production tax redresses the slope of the wage curve.

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Introduction

The present paper explores the impact of heterogeneity on taxation of frictional labor markets. It introduces a dimension of heterogeneity that results in equilibrium mismatch between workers and firms.

The public finance literature has recently been extended to account for search frictions on the labor market. The determination of wages and unemployment in frictional environments affects public policies like redistribution taxes (Golosov et al., 2013; Hungerbühler et al., 2006) and unemployment insurance (Landais et al., 2010). Contrary to standard frictionless environments, search frictions may generate additional equilibrium inefficiencies when agents are heterogeneous (Shimer and Smith, 2001). Such cases are circumvented in the literature by assuming homogeneity or complete market segmentation.

As a central message, I find that heterogeneity weighs down the efficiency cost of redistribution or insurance. This result is proved rigorously within a particular limit-case framework, but it is likely to hold in more general settings. The economic mechanism relies on a general equilibrium effect which connects workers' search strategies to firms' decisions to create jobs. A job seeker, wishing to leave unemployment faster, will be *less selective*. This means that it may apply to jobs for which she is not the best qualified. Job seekers' selectivity determine the extent of mismatch on the labor market. Mismatch is thus an equilibrium outcome. From the firms' perspective, job mismatching deteriorates profits because jobs are less productive on average. Consequently, more mismatch leads to less job openings. By not internalizing job creation decisions, workers are not selective enough and thus job quality is diminished. Taxing high-productive jobs (or employed workers) thus reinforces this *composition externality* by reducing the gains from being selective.

This article builds a stylized model of the labor market with mismatch to derive simple optimal tax formulae. It focuses on the problem of efficiency when search efforts are unobservable, abstracting from equity concerns and risk aversion. Optimal taxation consists in a self-financing Pigouvian tax that maximizes total net output. The model is based upon three main hypotheses: random matching, segmentation of the labor market per jobs' type, and symmetric comparative advantages. Job mismatching does not arise without frictions on the labor market. The model adopts the framework of Diamond-Mortensen-Pissarides (see Pissarides 2000 for an overview), in which search frictions prevent instantaneous matching between firms and workers. As a departure from this standard model, the labor market is assumed to be segmented by firms' type. Workers choose a search effort for each type of firm and match randomly with employers. Such a segmentation of the labor market causes the composition externality. Lastly, heterogeneity is modeled *à la* Salop (1979) (see Marimon and Zilibotti (1999) and Decreuse (2008) for a similar setting). In this environment, workers have symmetric comparative advantages in producing across firms' types. As a consequence, the distance between a worker and her employer on Salop's circle is a sufficient statistic of the job's productivity. This modeling can be considered as an improvement from the model with homogeneous agents, and a first step towards a general account of two-sided heterogeneity.

This article presents two main results. Proposition 1 shows that there exists a payoff profile that decentralizes efficient job creation and efficient search efforts. Then, Proposition 2 shows that a particularly simple anti-redistributive taxation scheme can implement this payoff profile when wages are Nash-bargained.

The simple anti-redistributive taxation scheme can decentralize the first-best allocation even if (multi-dimensional) search efforts are not observed. This anti-redistributive taxation is simple because it only consists in a proportional tax on wages, a proportional tax on production and a lump-sum tax. It corrects for two inefficiencies of the labor market:

1. The proportional tax on wages maintains the Hosios-Pissarides condition (Hosios, 1990; Pissarides, 2000), which states that the match surplus is split according to the search externalities created by each side of the market (Boone and Bovenberg, 2002).

2. The proportional tax on production makes the wage curve steeper. As a consequence, job seekers internalize the composition externality by applying more selectively.

The optimal tax rates are independent of the search technology, the matching function and the production function. Regarding other public policies, neither unemployment benefits, a minimum wage nor a subsidy to job creation can decentralize the first-best allocation. In particular, unemployment benefits have ambiguous effects. They make workers more selective but they also make them search less for any kind of jobs.

A simulation of the model measures the welfare gap between the social optimum and the decentralized equilibrium without taxation. The introduction of heterogeneity to a benchmark search and matching model significantly increases the welfare gap. The welfare gap qualitatively depends the most on the workers' bargaining power. The lower the bargaining power, the less selective job seekers are. The presence of mismatch, however, cannot invalidate the findings of Hornstein et al. (2011) that search frictions only explain a low part of residual wage inequalities.

This paper is related to the public finance literature that features search frictions. Golosov et al. (2013); Schaal and Taschereau-Dumouchel (2012) and Hungerbühler et al. (2006) study optimal (second-best) redistribution through income taxation and unemployment benefits with different wage settings. Landais et al. (2010) accommodate the optimal unemployment insurance theory to frictional labor markets. These articles make assumptions that preclude mismatches. My paper focuses on optimal taxes in presence of inefficient mismatching, but abstracts from redistribution and insurance concerns. Several papers interest in the impact of income taxation on a frictional labor market, either in a positive approach (Pissarides, 1985, 1998; Lockwood and Manning, 1993) or in a normative approach (Boone and Bovenberg, 2002). The present article features job mismatching as a new dimension while preserving a simple and intuitive tax result.

This paper also belongs to the literature on the efficiency of markets with search frictions. The pioneer work of Mortensen (1982) demonstrates that the sharing of the joint production in the mating game affects the players' incentives to exert efforts. Players do not in general internalize the externality of their efforts on the others. Shimer and Smith (2001) deal with heterogeneous agents who choose a search intensity and a matching strategy. The high-productive agents do not search enough compared to the low-productive ones who search too much. The idea that "agents do not aim high enough" is present in my paper and recurrent in articles featuring either heterogeneous agents (Lockwood, 1986; Uren, 2006; Decreuse, 2008) or specific investments (Acemoglu and Shimer, 1999; Acemoglu, 2001; Charlot and Decreuse, 2005; Amine and Santos, 2008). The paper models mismatch as Decreuse (2008) with intensive industry-specific search efforts.[1] At the decentralized equilibrium, too much search effort is devoted to low-paid rather than high-paid jobs. Although search is directed, the model does not satisfy the efficiency property of the competitive search framework (Moen, 1997; Menzio and Shi, 2011) because firms are not allowed to post worker-specific job offers. Instead, wages are Nash-bargained when an employer and an employee meet. With this wage setting, Acemoglu (2001) and Amine and Santos (2008) highlight the effect job seekers' selectivity can have on firms' technological choices. In this paper, instead, search decisions do not affect the production technology but the quantity of jobs opened by firms.

The structure of the paper is as follows. The first section defines the theoretical framework and proves the two propositions. The second section explores the impact of taxation on the decentralized equilibrium relative to the first-best allocation with a positive approach. The model is then calibrated in a fourth section. Section 5 discusses extensions regarding risk-aversion and heterogeneity. I conclude in the sixth section.

1 The search and matching framework

This section introduces the theoretical model. First, notations and the main hypotheses are presented. Second, the first-best allocation is characterized. In the third part, the decentralized equilibrium is defined and the two propositions are stated.

1.1 The environment

Time is continuous. The economy is populated by infinitely-lived risk-neutral heterogeneous workers. Each worker has a fixed type $i \in I$. The distribution of worker types is

[1] Decreuse (2008) considers the extensive margin of search and discusses the intensive case in an extension section.

exogenous and the mass of workers is normalized to one. At any time, a worker can be employed in an industry $j \in J$ where she produces y_{ij}, or she can be unemployed and consumes home production h. u_i denotes the measure of i-type unemployed workers. A worker must be employed by a firm to produce. To employ a worker in a particular industry, a firm must open an industry-specific job vacancy and pay a cost k until the vacancy is filled. The distribution of firm types is thus endogenous. As the production technology exhibits constant returns to scale, multiple-job firms are not different from a collection of single-job firms. A firm will thus refer to a single job position, filled or vacant. Firms and workers discount future at rate r and the economy is at steady state.

Search and matching Search frictions prevent the instantaneous matching of job vacancies and workers. Search activities are on workers' own initiatives. A worker can observe the firms' type but cannot distinguish between firms of a given type. Consequently, I assume that each industry j constitutes an independent segment of the labor market. Workers are able to direct their search towards particular segments or industries. On each segment j, workers of any type can only meet a j-type firm. The meeting probabilities within a segment are governed by a constant-returns-to-scale function in the tradition of Diamond-Mortensen-Pissarides.[2] This segmentation of the labor market with independent matching probabilities is standard in directed search models (Moen, 1997; Menzio and Shi, 2010). The two following assumptions are less standard. First, workers decide on industry-specific search efforts as an intensive margin. Second, contrary to the majority of directed search models, wages are not posted by firms but Nash-bargained when a firm and a job applicant meet. Nash bargain is a crucial hypothesis in generating inefficiencies. Nevertheless, wage-posting, the alternative wage setting, may be controversial given worker heterogeneity. Wage-posting would require each firm to engineer (and commit to) a wage contract conditional on each type i of job applicants, before any meeting. We discuss the wage-posting setting in the penultimate section.

By incurring a search cost $e \geq 0$, a job seeker multiplies his baseline job-finding rate m_j on any segment j by a search intensity $s(e)$, where $s : \mathbb{R}^+ \to \mathbb{R}^+$ is twice differentiable,

[2] See Pissarides (2000) for an overview.

increasing and concave. The search technology has decreasing returns to scale at the segment level: the more a worker searches on a particular segment, the less she multiplies her job-finding rate. No effort produces a nil search intensity, $s(0) = 0$, and the marginal search intensity tends to infinity and to zero respectively as the effort tends to zero and to infinity, $\lim_{e \to 0} s'(e) = \infty$ and $\lim_{e \to \infty} s'(e) = 0$.[3] A search strategy consists in choosing a search effort (expressed in cost units) for each segment of the labor market. For the sake of simplicity the focus is on the symmetric case in which same-type workers choose the same search strategy. i-type unemployed workers choose a search effort e_{ij} for any segment j in J.

The baseline job-finding rate m_j is endogenously determined by a matching technology. On the segment j, the measure of applicants is $\int_I s(e_{ij}) u_i di$, expressed in efficiency units. The number of vacancies is denoted v_j. The measure of new meetings on a segment of the labor market is $M\left(\int_I s(e_{ij}) u_i di, v_j\right)$ with M a Cobb-Douglas function with constant returns to scale. The elasticity of the meeting function relative to the measure of employers is denoted η satisfying $0 < \eta < 1$. The baseline job-finding rate m_j is a function of industry-specific market tightness θ_j, i.e. the ratio of the employers' mass to the efficient job seekers' mass,

$$\theta_j \equiv \frac{v_j}{\int_I s(e_{ij}) u_i di}, \qquad\qquad m_j = m(\theta_j) \equiv M(1, \theta_j).$$

Consequently, the function m is differentiable, strictly increasing and concave from $\mathbb{R}^+$ onto $\mathbb{R}^+$ and $\eta = \frac{\theta_j m'(\theta_j)}{m(\theta_j)}$. A i-type worker exerting an effort e_{ij} on segment j meets a firm at the Poisson rate $s(e_{ij}) m(\theta_j)$. Conversely, a j-type firm meets a i-type worker at the rate

$$\frac{s(e_{ij}) u_i}{\int_I s(e_{i'j}) u_{i'} di'} \frac{m(\theta_j)}{\theta_j}.$$

This expression derives from the equality between the flow of workers i meeting a firm j and the flow of firms j meeting a worker i. As a consequence, $q(\theta_j) \equiv \frac{m(\theta_j)}{\theta_j}$ is the rate

[3] The model readily accommodates for match-specific search technology, namely s as function of both the effort and the match (i, j). For instance, a same effort e could further improve the job-finding rate for job seekers more productive in the industry. The results for efficiency and optimal taxation are unaffected.

at which a firm j meets a worker unconditional on her type. Workers can anticipate the job-acceptance decision, meaning the choice of each party to agree or not on matching after a meeting. They will never rationally pay a positive search cost for unsuccessful meetings. In other words, the optimal search strategy internalizes the job-acceptance margin; any meeting leads to a match. Once matched, a firm j pays an i-type employee an after-tax transfer w_{ij} until the job breaks at an exogenous rate δ, independent of the job's productivity. The wage is Nash-bargained and β defines the bargaining parameter of workers with $0 \leq \beta < 1$.

One-dimension heterogeneity The results of the paper rely on a particular modeling of two-sided heterogeneity. In the spirit of Salop (1979), the type sets I and J are both isomorphic to the unit circle, and y_{ij} only depends on the (shortest) arc-distance x_{ij} between i and j on the unit circle.[4] Define the production function $y : (0, 1/2) \to \mathbb{R}^+$ such that $y_{ij} = y(x_{ij})$. y is differentiable and strictly decreasing. Workers are less productive when matched with a more distant firm. A (i, j)-type job reaches the highest productivity when $i = j$ ($x_{ij} = 0$), and the lowest productivity when i and j are opposite on the circle ($x_{ij} = 1/2$). A worker's type interprets as her endowment in specific skills. Symmetrically, a firm's type represents its requirements in skills. The productivity of each job depends on the match between the employee's skills and the firm's requirements. This assumption reduces the dimensionality of heterogeneity from two to one.

In addition, assume that worker types are uniformly distributed on the unit circle. Comparing to unconstrained two-sided heterogeneity, the current model removed absolute advantages. Although two workers may have different types, they cannot be ranked by skills or abilities. Workers merely differ in the industries in which they are productive. This assumption may miss several important aspects of empirical labor markets. Such a modeling, however, is an improvement comparing to models without heterogeneity or job mismatching. Here, the labor market exhibits mismatch and the analysis remains tractable and comparable to the benchmark with homogeneous agents. Even if workers

[4] Types are continuously distributed on the unit circle, but one can also consider a discrete distribution of equally-spaced types on the unit circle. The results require a symmetric structure.

are uniformly distributed and same-type workers play the same search strategy, there may exist equilibria in which market tightness θ_j differs across segments. To obtain symmetry and so tractability, the analysis focuses on equilibria for which θ_j is constant over j, as Decreuse (2008) does. Accordingly, notations can be simplified. Any match-specific variable A_{ij} solely depends on the distance $x = x_{ij}$. It can be denoted A_x. $x \in (0, 1/2)$ now refers to the match type. Any worker-specific (industry-specific) variable A_i (A_j) is constant over i (j) and can be simply denoted A without indexes. In particular, θ denotes market tightness for each segment $j \in J$, and e_x is the search effort of an unemployed worker of any type i providing that the prospected industry j satisfies $x = x_{ij}$.

Denote a search strategy $\mathbf{e} = \{e_x\}_{x\in(0,1/2)}$. The operator $\mathbb{E}_{x|\mathbf{e}}$ is defined such that

$$\mathbb{E}_{x|\mathbf{e}} A_x \equiv \int_0^{1/2} \frac{s(e_x)}{\int_0^{1/2} s(e_{x'}) 2dx'} A_x 2dx.$$

$\mathbb{E}_{x|\mathbf{e}}$ interprets as the expected value of A_x over the distribution of workers employed in any industry j at steady state. In particular, $\mathbb{E}_{x|\mathbf{e}} y_x$ is the mean productivity of jobs in any industry. This distribution is an endogenous outcome as it depends on the search strategy.

Tax instruments The model is completed by taxes. The fiscal authority does not have access to full information. It is able to observe productions and wages but not search efforts. It cannot either control the wage bargaining nor the entry decision of firms. Accordingly, define the production tax rate τ^y, the payroll tax rate τ^w and the lump-sum tax τ^f. Flow profits of a match x is

$$\pi_x \equiv (1 - \tau^y) y_x - (1 + \tau^w) w_x - \tau^f.$$

Note that the production tax can be substituted by a profits tax without loss of generality. Define $T_x \equiv \tau^y y_x + \tau^w w_x + \tau^f$, the amount of taxes levied on a match x. The tax system is self-financing, meaning the fiscal authority is constrained to make no benefits and no losses, $\mathbb{E}_{x|\mathbf{e}} T_x = 0$. A production tax is unusual in the literature on taxation and frictional labor markets. We will show that this tax instrument is necessary to decentralize the first-best optimum. In section 4, we study other tax instruments such as unemployment benefits and non-linear taxes.

Definition 1 *A steady-state allocation is*

- *a distribution of agents* $\mathcal{D} = (\{n_x\}_{x\in(0,1/2)}, u, v)$*, where* n_x *is the measure of* x*-type jobs,* u *the measure of unemployed workers,* v *the measure of job vacancies;*
- *and a payoff profile* $\mathcal{P} = \{w_x, \pi_x\}_{x\in(0,1/2)}$*, where* w_x *and* π_x *are the payoffs to the worker and to the employer in a* x*-type match.*

  An allocation is feasible if and only if the mass of workers is one ,

$$\int_0^{1/2} n_x 2dx + u = 1,$$

  and total output exceeds vacancy costs and transfers

$$\int_0^{1/2} (\pi_x + w_x)\, n_x 2dx + kv \leq \int_0^{1/2} y_x n_x 2dx.$$

1.2 First best: efficient matching

Consider the problem of a benevolent utilitarian social planner who must choose i) the number of job vacancies to create, or equivalently market tightness θ, and ii) the search strategies $\mathbf{e} = \{e_x\}_{x\in(0,1/2)}$. Any solution is defined as a first-best optimum. The optimal allocation without search frictions, which exhibits full employment and no mismatch, is not considered. The social planner only cares about efficiency because workers are risk-neutral. It maximizes total discounted net output $\int_0^\infty Y_t \exp(-rt)dt$, with net output Y_t defined as

$$Y_t \equiv \int_0^{1/2} y_x n_x 2dx + hu - kv - \left(\int_0^{1/2} e_x 2dx\right) u.$$

Net output is the sum of four terms: employment production, home production, the vacancy costs and the search costs. The measures of jobs n_x, vacancies v and unemployed workers u are time-varying but the time index is dropped for clarity. They enter the problem as constraints of the optimization program:

$$\begin{cases} \dot{n}_x = s(e_x)m(\theta)u - \delta n_x, \qquad \forall x \in (0, 1/2) \\ \dot{u} = \delta\,(1-u) - \left(\int_0^{1/2} s(e_x)m(\theta)2dx\right) u \\ v = \theta \left(\int_0^{1/2} s(e_x)2dx\right) u \end{cases}.$$

The first two constraints describe the dynamics of employment and unemployment. For a small time period Δt, a fraction $s(e_x)m(\theta)\Delta t$ of unemployed workers gets employed to form a x-type match. Simultaneously, a fraction δ of employed workers becomes unemployed. The last constraint is exactly the definition of the market tightness.

The next lemma is equivalent to Proposition 1 of Acemoglu and Shimer (1999), which characterizes the efficient allocation of their model with homogeneous workers. Following these authors, ψ denotes the social flow value of unemployment, so that one more unemployed worker increases steady-state output by ψ/r. ψ is directly comparable to a productivity level and can be interpreted as the opportunity cost of being employed. The planner's problem reduces to a simple form.

Lemma 1 *The planner's problem characterizes the relative value of unemployment, ψ, as*

$$\psi = h + \int_0^{1/2} s(e_x)m(\theta)\left[\frac{y_x - \psi}{r+\delta}\right] 2dx - k\theta \int_0^{1/2} s(e_x)2dx - \int_0^{1/2} e_x 2dx, \qquad (1)$$

and the optimization program is equivalent to maximizing the value of unemployment ψ in (1) *with respect to θ and e_x for all $x \in (0, 1/2)$.*

A proof is given in appendix A. Note that maximizing ψ as an explicit function of $(\theta, \mathbf{e})$ and maximizing the recursive (implicit) definition of ψ, the right-hand side of (1), are equivalent thanks to the envelop condition. The value of unemployment accounts for home production (first term). The second term discounts the perspectives of future employment. At a rate $s(e_x)m(\theta)$, the unemployed worker finds a job yielding the present-discounted net gain $\frac{y_x-\psi}{r+\delta}$. Jobs are more socially valuable when the discount rate and the separation rate are lower because individuals are more patient and jobs last longer. The two negative terms represent the vacancy cost per unemployed worker and the individual search cost.

We apply Lemma 1 and derive the first-order conditions for θ and $\mathbf{e}$. A classical trade-off arises when fixing market tightness. On the one hand, workers match and start producing more rapidly when the labor market is tight for firms ("thick-market externality"). The second term in the value of unemployment (1) is increasing in θ. On the other hand, firms wait longer before meeting a worker and so they have to pay a higher expected vacancy cost ("congestion externality"). The third term is decreasing. The first-order condition relative to market tightness θ can be written as the balance between these negative and positive externalities at the margin (within each segment of industry),

$$\frac{k}{q(\theta)} = \eta \left[\frac{\mathbb{E}_{x|\mathbf{e}} y_x - \psi}{r + \delta} \right]. \tag{2}$$

The expected vacancy cost on the left-hand side must be equal to the share η of the expected match surplus, meaning the expected job productivity net of the unemployment value. The efficient search intensity satisfies the following first-order condition:

$$\begin{cases} e_x = 0 & \text{if } \frac{y_x - \psi}{r+\delta} - \frac{k}{q(\theta)} \leq 0 \\ s'(e_x) m(\theta) \left(\frac{y_x - \psi}{r+\delta} - \frac{k}{q(\theta)} \right) = 1 & \text{else} \end{cases}. \tag{3}$$

Jobs whose production is not high enough to compensate the value of unemployment and the expected vacancy cost together are not prospected. If the productivity of the worst feasible matches is too low, no search effort is exerted beyond a threshold distance in the skills space. This is true if a match at the other side of the Salop's circle has zero-productivity, $y(1/2) = 0$, for instance. As long as the net value of a match is positive, searching is efficient because the marginal gain from searching will be infinite at a zero level of efforts. The efficient strategy consists in searching until the marginal gain equalizes the marginal cost. As the marginal cost is 1 for any segment of the labor market, the marginal gains from searching equalize across segments. The optimal search strategy e_x is decreasing in x. In other words, workers have to search more for jobs whose productivity is higher.

Definition 2 *A first-best allocation is characterized by a value of an unemployed worker* ψ*, market tightness* θ *and a search strategy* $\mathbf{e}$ *fulfilling* (1)*,* (2) *and* (3)*.*

The distribution of agents $\mathcal{D}$ derives from the steady-state definitions of n_x, u and v. Note that any feasible payoff profile $\mathcal{P}$ is compatible with an efficient-matching allocation because agents are risk-neutral. Consequently, redistribution does not change welfare. In appendix, we show that the distribution of agents in an efficient allocation is unique.

For the same reasons as Shimer and Smith (2001), some mismatch may be efficient under search frictions. By being more selective, a worker pays more search costs but also makes firms wait longer before matching. It may be efficient for a worker to apply to jobs that are not their best fit ($x = 0$) in order to increase the job-filling rate of firms.

1.3 Decentralized equilibrium

Now, let us consider the decisions of workers and firms regarding search activities and job openings. Let W_x be the worker present-discounted value of a x-type match. U is defined as the asset value of being unemployed. This quantity is the worker's surplus, once the job quality y_x is revealed to the worker and the firm. When a worker is unemployed, she consumes home production and incurs search costs. She obtains a x-type job at the rate $s(e_x)m(\theta)$ and makes a capital gain $W_x - U$. The asset value of unemployment is accordingly defined by

$$rU = h + \int_0^{1/2} s(e_x)m(\theta)\left[W_x - U\right] 2dx - \int_0^{1/2} e_x 2dx. \tag{4}$$

An employee earns the net-of-tax wage w_x. She may return to the pool of unemployment if her match breaks, which occurs at a rate δ. The asset value of a match satisfies

$$rW_x = w_x + \delta\left(U - W_x\right). \tag{5}$$

When matched, a worker misses the flow value of unemployment rU until the job destruction. By definition, $\psi \equiv rU$ in the decentralized equilibrium. Combining equations (4) and (5) provides an expression of the value of an unemployed worker:

$$\psi = h + \int_0^{1/2} s(e_x)m(\theta)\left[\frac{w_x - \psi}{r+\delta}\right]2dx - \int_0^{1/2} e_x 2dx. \tag{6}$$

This characterization of the value of unemployment differs from the social value defined in (1) in two respects. First, workers compare the expected wage to the value of unemployment, whereas the planner compares the expected output. Second, workers do not internalize the vacancy cost. Workers decide on their search strategy by maximizing their return in (6), The search effort then satisfies

$$\begin{cases} e_x = 0 & \text{if } w_x \leq \psi \\ s'(e_x)m(\theta)\left(\frac{w(x)-\psi}{r+\delta}\right) = 1 & \text{else} \end{cases}. \tag{7}$$

When the labor income w_x is lower than the returns to unemployment ψ, the unemployed workers do not exert any search effort.[5] If the value of a match from a worker's perspective is high enough, job seekers are willing to search until the marginal benefit equals 1.

From the perspective of the firms, J_x denotes the firm's asset value of a x-type match. Prior to meeting a worker, the expected value is $\mathbb{E}_{x|\mathbf{e}}J_x$. Firms create jobs until reaching zero profit at the steady state. The no-arbitrage condition for free entry is the equality between the expected cost of holding a vacancy and the expected value of filling a vacancy. It can be expressed

$$\frac{k}{q(\theta)} = \mathbb{E}_{x|\mathbf{e}}J_x. \tag{8}$$

For the firm's side, the *ex-post* surplus of a x-type match, J_x, must be distinguished from the *ex-ante* surplus, $J_x - \frac{k}{q(\theta)}$. The no-arbitrage condition restates as the expected ex-ante surplus to be equal to zero. When the employer meets a potential employee, however, she

[5] Actually, workers do not exert any effort either if $\pi_x < 0$ because the match would be rejected by the firm. It is implicitly assumed that $\pi_x > 0$ when $w_x > \psi$ for the exogenous wage profile. Nash-bargained wages satisfy this condition.

accepts the match as long as the ex-post surplus is positive, $J_x > 0$. The vacancy cost is sunk before the meeting, and so it is not accounted for in the job-acceptance decision. In absence of precision, the surplus refers to the ex-post surplus as in the literature. The ex-ante surplus will always be accurately named. A match yields a net flow profit π_x to the employer, with a risk to be broken at a rate δ,

$$rJ_x = \pi_x - \delta J_x. \tag{9}$$

From equations (8) and (9), the total expected vacancy cost is equal to the present-discounted value of expected profits:

$$\frac{k}{q(\theta)} = \frac{\mathbb{E}_{x|\mathbf{e}}\pi_x}{r+\delta} \tag{10}$$

Higher profits attract more firms so market tightness increases with expected profits.

1.3.1 Optimal wage profile

Consider the problem of a benevolent utilitarian social planner who let agents determine themselves market tightness θ and search efforts $\mathbf{e}$, but who controls payoffs π_x and w_x for any x. Can the social planner decentralize the first-best allocation with appropriate transfers? We prove here that it can.

Definition 3 *Given a payoff profile $\mathcal{P}$, a decentralized equilibrium with exogenous transfers is characterized by value of an unemployed worker ψ, market tightness θ and a search strategy $\mathbf{e}$ fulfilling equations (6), (7) and (10).*

The comparison with a first-best allocation yields the following Proposition.

Proposition 1 *A decentralized equilibrium with exogenous transfers is efficient if and only if the payoff profile $\mathcal{P} = \{\pi_x, w_x\}_{x\in(0,1/2)}$ is such that firms obtain a share η of the match surplus on expectation over the distribution of match types,*

$$\mathbb{E}_{x|\mathbf{e}}\pi_x = \eta\mathbb{E}_{x|\mathbf{e}}\left[y_x - \psi\right], \tag{11}$$

and such that the production of each match covers exactly the after-tax labor incomes and the vacancy cost:

$$\frac{y_x}{r+\delta} = \frac{w_x}{r+\delta} + \frac{k}{q(\theta)} \qquad \forall x \in (0, 1/2). \tag{12}$$

The first condition requires the match surplus to be split $\eta/1-\eta$ between firms and workers on expectation. The match surplus should be split according to the ability of each side to create positive versus negative search externalities, depending on the meeting function. When the elasticity of the matching function η is high, firms are more efficient than workers in the search process and consequently deserve higher gains from matching. The congestion externality (negative) they impose on the other firms, which want to fill their vacancy too, is lower and the thick-market (positive) externality on job seekers is higher. Symmetrically, when η is low, job seekers produce better externalities and so they should receive a higher share of the surplus.

The second equation stipulates that profits must cover the vacancy cost for each match x. In particular, the slope of the wage curve with respect to productivity, $\frac{\partial w_x}{\partial y_x}$, must be equal to one. This condition is always true on expectation, given equation (10) and the self-financing condition, $\mathbb{E}_{x|\mathbf{e}}\pi_x = \mathbb{E}_{x|\mathbf{e}}y_x - \mathbb{E}_{x|\mathbf{e}}w_x$. Condition (12) may not hold because the vacancy cost is sunk before a match. For instance, a match may generate a positive ex-post surplus (and so is accepted) but may still have a negative ex-ante surplus.

Importantly, Proposition 1 can be generalized to unconstrained two-sided heterogeneity. The decentralized equilibrium can be a first-best allocation if firms receive a share η (possibly industry-specific η_j) of expected surplus and production covers wages and vacancy costs for each pair (i, j). The assumption of symmetric comparative advantages is necessary for Proposition 2: the social planner can implement the optimal payoff profile with a simple tax scheme.

1.3.2 Optimal taxes with Nash bargain

Consider now that agents determine payoffs through Nash-bargaining. Agents determine market tightness θ and search strategy $\mathbf{e}$, but also wages and profits. The social planner can only chose the self-financing tax scheme (τ^y, τ^w, τ^f) to provide the right incentives. We prove here that the planner can decentralize the efficient-mismatch equilibrium.

Wages are solutions of the following maximization:

$$\max_{w_x} \; (W_x - U)^\beta \, J_x^{1-\beta} \text{ s.t. (5) and (9).}$$

The outside options of workers U is taken as fixed because it is not impacted by the individual agreement between the employer and the employee. The firm and the employee take the three components of taxation τ^y, τ^w and τ^f as given. Because the fiscal authority clears its budget, the tax parameters may actually depend on endogenous outcomes. This is a reasonable assumption in a model in which the firm's size is negligible compared to the labor market's size and collusion is excluded. Profits are decreasing with wages and the objective function is concave by assuming $1 + \tau^w > 0$. Otherwise, the firm would benefit from setting the highest wage possible.

To clear the budget constraint, the fiscal authority looses one degree of freedom. We interpret τ^f as determined by $\mathbb{E}_{x|\mathbf{e}} T_x = 0$, which writes as $\tau^f = -\tau^y \mathbb{E}_{x|\mathbf{e}} y_x - \tau^w \mathbb{E}_{x|\mathbf{e}} w_x$. The tax scheme non-trivially alters the wage setting.

Lemma 2 *When the fiscal authority clears its budget, $\mathbb{E}_{x|\mathbf{e}} T_x = 0$, bargained after-tax wages satisfy the following properties:*

- *the expected wage is a weighted average of the expected productivity and the opportunity cost of employment,*

$$\mathbb{E}_{x|\mathbf{e}} w_x = \beta^w \mathbb{E}_{x|\mathbf{e}} y_x + (1 - \beta^w)\psi, \tag{13}$$

where β^w is the worker's share of the surplus defined by

$$\beta^w \equiv \frac{\beta}{1 + \tau^w - \beta \tau^w};$$

- *the relative wage is proportional to the relative job's production,*

$$w_x - \mathbb{E}_{x|\mathbf{e}} w_x = \beta^s \left(y_x - \mathbb{E}_{x|\mathbf{e}} y_x \right), \tag{14}$$

where β^s is the slope of the wage curve defined by

$$\beta^s \equiv \frac{\beta(1-\tau^y)}{1+\tau^w}.$$

In the laissez-faire, both the worker's share of the surplus β^w and the slope of the wage curve β^s are equal to the bargaining parameter β.

The proof of the lemma is in appendix C. The first part of the lemma is not new in the literature. The match surplus $J(x)+W(x)-U$ depends on the wage as long as the payroll tax τ^w is nonzero. Agents internalize this effect when they bargain the wage. If the tax rate is positive, an employee has to bargain more aggressively to obtain the same wage without taxation because her employer incurs a higher cost of labor (the before-tax wage). This is equivalent to a decline in the worker's share of the surplus, thus β^w is decreasing in τ^w. The effect of payroll taxes on wage bargaining is emphasized by Boone and Bovenberg (2002); Lockwood and Manning (1993); Pissarides (1985, 1998). The choice of a wage for the employer-employee match does not affect the amount of production taxes paid nor the lump-sum component. Hence the production tax rate τ^y does not influence surplus sharing and does not appear in the definition of β^w.

The second part of the lemma is original. Given that the tax scheme is self-financing, a change in one of the two tax rates τ^y and τ^w has redistributive effects. When the tax rates are positive, the lump-sum tax is negative so that the fiscal authority makes zero profits. In that case, high-productive jobs are more taxed than low-productive jobs, the latter are actually subsidized through the lump-sum component of taxation. The wage curve is flatter relative to *laissez-faire*, or equivalently β^s is diminished. Taxation can decentralize the first-best allocation because it can disentangle the two roles played by the bargaining parameter β.

By incorporating the surplus-sharing rule in the no-arbitrage condition for free entry

(10), equilibrium market tightness can be obtained as a function of the search strategy and the value of an unemployed worker,

$$\frac{k}{q(\theta)} = (1-\beta^w)\left[\frac{\mathbb{E}_{x|\mathbf{e}}y_x - \psi}{r+\delta}\right]. \tag{15}$$

The job creation decision of firms defines market tightness as a function of search strategy $\mathbf{e}$ and the value of unemployment ψ. Firms decide to open new vacancies until the total expected vacancy cost equals their share of the total match surplus. The higher their share $(1-\beta^w)$, the higher will be the number of job vacancies per unemployed. Substitute the expected wage in equation (14) using (10) to get

$$w_x = \beta^s y_x + (1-\beta^s)\mathbb{E}_{x|\mathbf{e}}y_x - (r+\delta)\frac{k}{q(\theta)}.$$

By substituting this expression in equation (6), we obtain equation (1), meaning the same recursive definition of ψ as the efficient allocation. By substituting the wage in equation (7), search efforts satisfy

$$\begin{cases} e_x = 0 & \text{if } \frac{\beta^s y_x + (1-\beta^s)\mathbb{E}_{x|\mathbf{e}}y_x - \psi}{r+\delta} \leq \frac{k}{q(\theta)} \\ s'(e_x)m(\theta)\left(\frac{\beta^s y(x) + (1-\beta^s)\mathbb{E}_{x|\mathbf{e}}y_x - \psi}{r+\delta} - \frac{k}{q(\theta)}\right) = 1 & \text{else} \end{cases}. \tag{16}$$

There are in general several best strategies $\mathbf{e}$ satisfying (16) given market tightness θ and the value of unemployment ψ.[6] The search effort e_x is decreasing in the distance x. Job seekers exert more search efforts for high-productive jobs.

Definition 4 *A decentralized equilibrium with taxes and Nash bargain is characterized by value of an unemployed worker ψ, market tightness θ and a search strategy $\mathbf{e}$ fulfilling equations* (1), (15) *and* (16).

A comparison with the first-best allocation shows that satisfying both $\beta^w = 1-\eta$ and

[6] One can check that $\frac{\beta^s y_x + (1-\beta^s)\mathbb{E}_{x|\mathbf{e}}y_x}{r+\delta} - \psi - \frac{k}{q(\theta)} > 0$ and $\pi_x > 0$ are equivalent. A worker and a firm always agree on matching.

$\beta^s = 1$ is a sufficient condition for any decentralized equilibrium to be efficient. The next proposition states it as a necessary condition. This is the key result of the paper.

Proposition 2 *A decentralized equilibrium with Nash bargain is efficient if and only if*

- *the worker's share of the surplus β^w is equal to the elasticity of the matching function with respect to the number of applicants $1-\eta$,*

$$\beta^w = 1 - \eta;$$

- *and the slope of the wage curve β^s is equal to 1,*

$$\beta^s = 1.$$

The optimal taxation scheme is such that

- *the tax rates satisfy*

$$\tau^{w*} = -\frac{1-\beta-\eta}{(1-\beta)(1-\eta)}, \tag{17}$$

$$\tau^{y*} = \frac{1-\beta-\eta}{(1-\beta)(1-\eta)} - \frac{\eta}{1-\eta}; \tag{18}$$

- *the lump-sum tax is always positive,*

$$\tau^{f*} = -\tau^{y*}\mathbb{E}_{x|\mathbf{e}}y_x - \tau^{w*}\mathbb{E}_{x|\mathbf{e}}w_x > 0;$$

- *the fiscal authority debits the following amount for an x-type job:*

$$T_x^* = -\frac{\eta}{1-\eta}(y_x - \mathbb{E}_{x|\mathbf{e}}y_x).$$

The optimal tax scheme is anti-redistributive, meaning low-productive jobs are taxed and high-productive jobs are subsidized.

Proof. We show that $\beta^w = 1-\eta$ and $\beta^s = 1$ are necessary conditions. Suppose there exists β^w and β^s such that a decentralized equilibrium $(\psi, \theta, \mathbf{e})$ is efficient. Therefore, $(\psi, \theta, \mathbf{e})$ satisfies simultaneously (1), (2), (3), (15) and (16). Fulfilling simultaneously (2) and (15) imposes $\beta^w = 1-\eta$. Fulfilling simultaneously (3) and (16) imposes $\beta^s = 1$. ■

The bargaining parameter has two incompatible objectives to satisfy. It must satisfy the Hosios-Pissarides conditions so that firms and workers share the match surplus according to the externalities they impose on other actors.[7] These externalities are captured in the elasticity of the matching function η. Under this condition, firms create an optimal *quantity* of jobs. The bargaining parameter must also be equal to 1, so that workers fully internalize the cost of vacancies. Under this condition, workers search efficiently, leading to an optimal *quality* of jobs in terms of average productivity. The simple tax scheme corrects these two inefficiencies by disentangling the two roles of the bargaining parameter.

The second part of the Proposition characterizes the optimal taxation scheme. The wage tax is the only policy tool that affects surplus sharing. The optimal rate τ^{w*} corrects for any gap between the bargaining power of firms $1-\beta$ and the elasticity of the matching function η. If this gap is negative, $1-\beta < \eta$, the optimal tax rate will be positive to improve the firms' share of the surplus $1-\beta^w$ and to adjust it to η. Boone and Bovenberg (2002) find the same tax rate on wages to restore the Hosios-Pissarides condition. The wage tax modifies the slope of the wage curve but is fixed by the previous condition. The tax on production is the only instrument left to adjust the slope to 1. The tax rates skyrocket in absolute value when the elasticity of the matching function is high. In that case, firms create good search externalities on the labor market. They should obtain a high share of the surplus, or equivalently workers should have low bargaining power, so that an optimal quantity of jobs is created. On the other hand, fixing a low bargaining power reduces the selectivity of job seekers. Consequently, when the policy maker sets a high wage tax to improve the surplus sharing, it amplifies the composition externality

[7] See Hosios (1990) and Pissarides (2000).

which requires a stronger taxation on production.

Proposition 1 is remarkable for two reasons. First, the tax scheme is particularly simple, made of linear and lump-sum taxes. Even if the fiscal authority cannot observe search efforts, it is able to decentralize the first-best optimum. Second, the optimal tax rates do not depend on the production technology nor the search technology. In particular, multiple equilibria is not an issue. Any decentralized equilibrium can be made efficient.

This result is robust to any specification of the search technology. In the paper, the optimal search strategies are explicitly formulated through a first-order condition thanks to differentiability and convexity assumptions. These assumptions are made for convenience but are not necessary for the result. As long as job seekers can be choosy, meaning they still have the possibility of not visiting some segments of the labor market at no cost, the Proposition holds. The search technology may even be non-continuous. Indeed, the social planner's problem is identical to the workers' under optimal fiscal rules. The optimal policy not only decentralizes the solution of the planner's problem but the problem itself.

The optimal tax parameters may be negative or positive, depending on the value of the bargaining power β and the elasticity of the matching function η. They can never both be simultaneously positive. Otherwise, anti-redistribution would be impossible. Using optimal tax rates and the wage in (13), the lump-sum component is shown to be positive

$$\tau^{f*} = \frac{\eta}{(1-\beta)(1-\eta)} \left[\eta(\mathbb{E}_{x|\mathbf{e}} y_x - \psi) + (1-\beta)\psi\right] > 0.$$

The more productive a job is, the more it is subsidized. This idea is close to other results in the literature. Cahuc and Laroque (2014) study the case of monopsony on the labor market with heterogeneous workers. The optimal taxation requires jobs to be subsidized as the firm pays higher wages. Holmstrom (1982) focuses on the moral hazard problem of a principal with multiple agents. To avoid free-riding, the manager pays a bonus when the output is above a certain threshold. The manager is the equivalent to the planner in the present paper.

In many models, the Hosios-Pissarides condition is sufficient for the *laissez-faire* equilibrium to be efficient. It is not the case here.

Corollary 1 *When the bargaining power satisfies the Hosios-Pissarides condition,* $\beta = 1-\eta$*, optimal taxation is defined by*

$$\tau^{w*} = 0,$$
$$\tau^{y*} = -\frac{\eta}{1-\eta}.$$

When the Hosios-Pissarides condition is met, the wage tax must be nil to keep an efficient level of jobs created. The production tax is negative to insure anti-redistribution.

2 Two sources of inefficiency

This section analyzes the divergence between efficient search decisions and equilibrium ones, supported by graphical illustrations. Parameters β^w and β^s impact the equilibrium through two different channels. Each corresponds to a particular source of inefficiency.

Lemma 3 *Compared to the social planner decisions,*

1. *firms do not create enough jobs if and only if* $1-\beta^w \leq \eta$*, when the search strategy and the value of unemployment are given;*
2. *workers are not selective enough leading to less productive jobs on average if and only if* $\beta^s \leq 1$*, when market tightness and the value of unemployment are given.*

This lemma derives from the comparison between equations (2) and (15), and between equations (3) and (16). The proof of the second property is in appendix D.

2.1 Efficient surplus sharing

Suppose the search strategy, $\mathbf{e}$, and the value of unemployment, ψ, to be predetermined. The first point of Lemma 3 states that the best response of firms leads to too slack (respectively too tight) a labor market when the firms' share of the surplus, $1-\beta^w$, is below (above) the threshold η.

Figure 1 illustrates the optimal surplus sharing in a $(\theta, \mathbb{E}_{x|\mathbf{e}} w_x)$ plane. Optimal market tightness maximizes the value of unemployment in (1). This condition is equivalent to maximize the utility of the unemployed workers (6) when constrained by the zero-profit condition (10). The zero-profit condition is a fixed decreasing curve in the $(\theta, \mathbb{E}_{x|\mathbf{e}} w_x)$ plane. The right-hand side of (6) defines a set of decreasing isoutility curves that do not cross each other. Note the isoutility curves have the same horizontal asymptote at level ψ. The graphical transcription of the constrained maximization consists in finding the isoutility curve that i) corresponds to the highest workers' utility and, ii) still crosses the zero-profit condition. The isoutility curve is therefore tangent to the zero-profit curve at optimal market tightness. This condition is met only if the surplus is shared according to the weights η and $1 - \eta$. If the worker's share of the surplus is too low for instance, $\beta^w < 1 - \eta$, then surplus sharing will be inefficient as illustrated on figure 2. The workers' utility can be increased by bargaining a higher equilibrium wage and by reducing market tightness, keeping profits unchanged.[8]

The first inefficiency, requiring the Hosios-Pissarides condition, is widely commented in the literature. It occurs in the standard Diamond-Mortensen-Pissarides framework with homogeneous agents and jobs. The second inefficiency, discussed now, is the cause of inefficient mismatch.

2.2 Efficient search strategy

Market tightness and the value of unemployment are fixed. As long as $\beta^s < 1$, workers make too much effort in searching for low-productive jobs to the detriment of high-productive jobs for any equilibrium search strategy. As a consequence, the (average) *quality* of jobs created is too low.

The Nash bargain prevents workers from accruing the full benefits of an increase in productivity. Imagine a worker can get a job of productivity y paid w instead of a job of productivity y' paid w'. The output gain, which is also the gain accounted by the social planner, is $y - y'$. The worker, however, gains $w - w' = \beta^s(y - y') < y - y'$. The cost of

[8] Moen (1997) draws similar graphs with his competitive search framework.

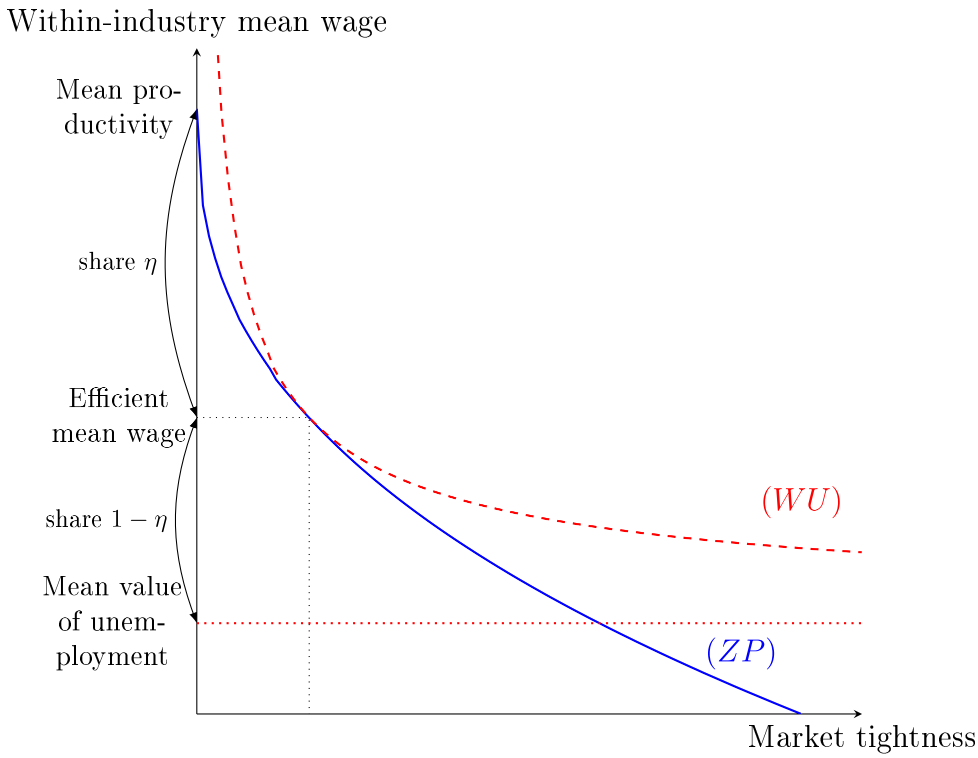

Figure 1: Optimal surplus sharing

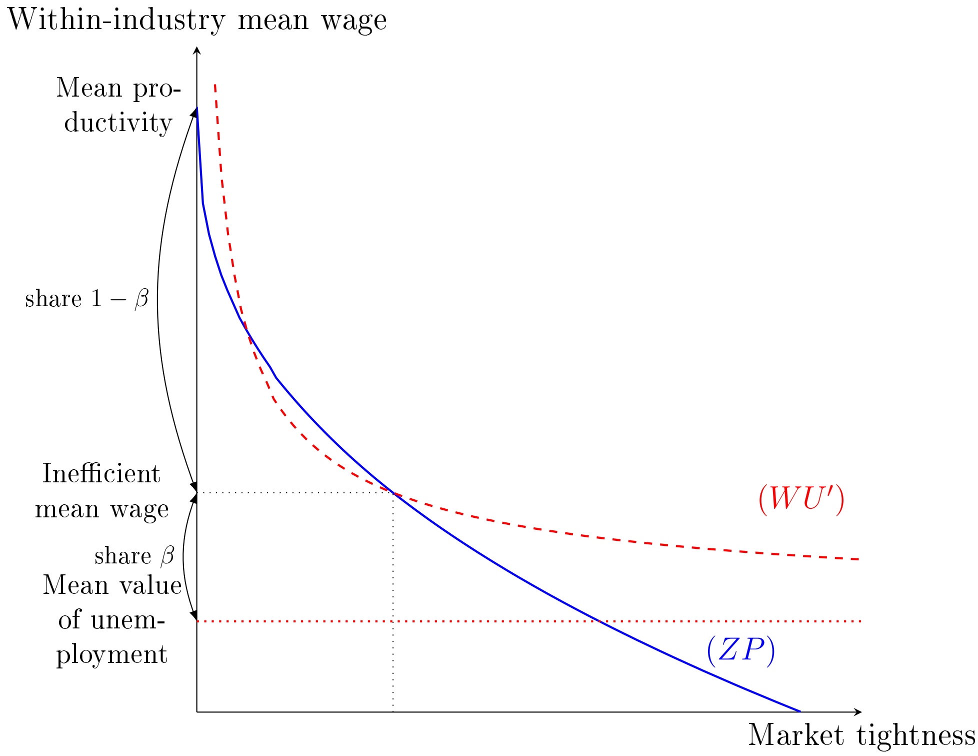

Figure 2: Inefficient surplus sharing, with $\beta^w < 1-\eta$

Note: the X-axis represents market tightness θ and the Y-axis the mean wage $\mathbb{E}_{x|\mathbf{e}}w_x$. The value of unemployment ψ and the search strategy $\mathbf{e}$ are fixed. The blue curves (ZP) correspond to the zero-profit condition for firms (10). The red dashed curves are isoutility curves on which a worker's utility, defined by the right-hand side of (6), is a constant. For any isoutility curve, the corresponding value of unemployment is also the level of the horizontal asymptote.

being selective, as an explicit search cost or as an opportunity cost to leave unemployment quicker, is fully incurred by workers or the social planner. Regarding the gains, workers are forced to share the benefits of their strategy with the firms through wage bargaining. In the decision to search for a productive job, a worker and the social planner evaluate the cost identically but the gain differently. Consequently, they do not make the same decision: the worker undervalues job quality. The equilibrium search strategy is thus inefficient in the *laissez-faire* scenario.

Figure 3 depicts the second property of Lemma 3 in an example. The social planner and workers evaluate the gains from searching differently. The two curves (OG) and (EG) represent the gains from matching respectively according to the efficient wage profile (optimum) and according to the decentralized wage bargaining (equilibrium). (OG) is defined by $z \mapsto \frac{z-\psi}{r+\delta} - \frac{k}{q(\theta)}$, and (EG) is defined by $z \mapsto \frac{\beta^s z+(1-\beta^s)\mathbb{E}_{x|\mathbf{e}} y_x - \psi}{r+\delta} - \frac{k}{q(\theta)}$. The tangent of angle between the two curves is $1 - \beta^s$. Given the search technology, the two different gain curves lead to two different search strategies, (OS) and (ES). Some low-productive jobs are prospected at equilibrium but should not be so. These jobs produce a positive ex-post surplus: the after-tax output is higher than the value of unemployment. Both the worker and the firm benefit from matching and can bargain a wage. These jobs still have a negative ex-ante surplus: the after-tax output is not high enough to compensate both the value of unemployment and the vacancy cost. Workers, knowingly, apply to these jobs. Even if firms have interest in rejecting such low-quality matches ex-ante, they lack a commitment device to make rejection a credible threat. Workers internalize the composition externality only when the slope of the wage curve, β^s, equals 1 as in the second condition of Proposition 1. The vacancy cost can be compensated for each match in this particular case. This is done graphically by superimposing the curves (EG) and (OG), leading to a superimposition of the curves (ES) and (OS) by construction. Whatever the production function and the search function (the black curves) are specified, the qualitative result remains: the optimal search strategy is always steeper than the equilibrium search strategy.

Note the second inefficiency, due to the composition externality, is close to the holdup problem, as formulated by Grout (1984). In holdup problems, workers (or firms) make an investment before matching on the labor market. As the cost of this investment is sunk before any meeting, workers (firms) under-invest for as long as they do not have full bargaining power. Search strategies can thus be compared to worker investments. The difference between the composition externality in this paper and the holdup problem, however, is that the former arises only with endogenous job creation. If the number of firms or jobs were fixed, the composition externality would disappear.[9]

3 Calibration

A calibration of the model measures of the welfare loss and the magnitude of optimal taxation. We can make two remarks before simulating the model. First, simulations are not necessary to show that the two proportional taxes can be high in absolute value. For example if $\beta = 1-\eta = 0.5$, then $\tau^{y*} = -100\%$. In this case, each job receives the (possibly negative) subsidy $y_x - \mathbb{E}_{x|\mathbf{e}} y_x$. The gain from having a better match is doubled as the surplus difference between two matches of types x and x' is $2\frac{y_x - y_{x'}}{r+\delta}$. Since the surplus sharing is 50-50 ($\beta^w = 0.5$), the difference $w_x - w_{x'}$ is exactly equal to $y_x - y_{x'}$ as desired.

Second, it is problematic that a decentralized equilibrium may not be unique in some circumstances. Intuitively, workers' selectivity and firms' job opening decisions can be strategic complement. When workers prospect too far from their best match, they decrease firms' profits and available jobs which reinforces the incentives to widen the search activities. In a simpler version of the model where search is an extensive margin, (weak) concavity of the production function is a sufficient condition to guarantee a unique equilibrium according to Decreuse (2008). In the subsequent numerical analysis, the production function is chosen linear and the calibrated models turn out to be unique. We do not show analytically, however, that concavity is a sufficient condition for uniqueness.

For the calibration strategy, we determine first a benchmark with standard calibrated parameters. The benchmark consists of the model with endogenous search but without

[9]See Acemoglu (1996, 2001); Acemoglu and Shimer (1999); Davis (2001); Masters (1998) for holdup problems in frictional labor markets.

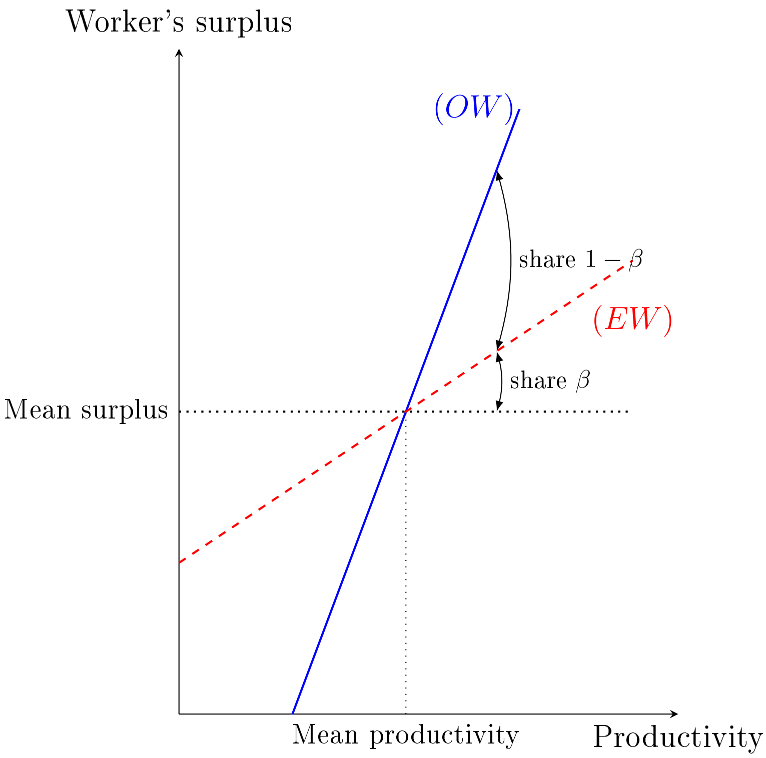

Figure 3: Inefficient vs. efficient search strategy

Note: market tightness θ and the value of unemployment ψ are fixed. The curve on the north-east part is the production function $y : x \mapsto y_x$. The dashed red curve (EG) corresponds to the mapping of any productivity level to the worker's capital gain. The blue curve (OG) represent the worker's capital gain under the efficient wage profile. On the south-west part of the graph, the black curve corresponds to the policy function linking gains to the exerted search efforts. This function is the same for the planner and private agents in equations (3) and (16). Lastly, (ES) and (OS) are the graphical representations of the equilibrium and optimal search strategies $x \mapsto e_x$.
Comment: the graph reads from the north-east part in the counterclockwise direction. The (EW) line overlaps with the (OW) line as β^s gets closer to 1.

heterogeneity. There are different segments $j \in J$ of the labor market but productivity is constant $y_x = 1$ and workers exert the same effort on each segment $e_x = e$. Then, we will depart from the benchmark with a non-constant productivity function.

The time scale is normalized so that one period is a quarter. We strictly follow the calibration of Shimer (2005), augmented by endogenous search efforts.[10] The discount rate r is chosen to match an annual interest rate of 5%. The job destruction rate is such that 10% of jobs break each quarter. The search function $s(.)$ is taken in the family of isoelastic functions. The elasticity of search intensity with respect to search cost is estimated by Christensen et al. (2005). Following Hornstein et al. (2011), I take it equal to 1. Multiplying the matching function $m(.)$ is equivalent to multiplying the search function $s(.)$ by the same coefficient. The matching function is normalized so that $m(1) = 1$, which implies $m(\theta) = \theta^{\eta}$. The multiplicative term of the search function is taken to match the job-finding rate of 1.35 in Shimer (2005). The elasticity of the matching function is taken at 0.25, and the bargaining power satisfy the Hosios-Pissarides condition $\beta = 1 - \eta$. Equilibrium market tightness θ is normalized to 1, which determines the value of the vacancy cost k. We obtain an equilibrium search cost e of the benchmark model at 0.27, which is of the same magnitude as the search costs estimated by Gautier et al. (2016). Lastly, home production is such that the flow payoff under unemployment, $h - e$, is equal to 0.4 as in Shimer (2005). Table 1 sums up the values chosen for the parameters. The equilibrium unemployment rate reaches 7.2%. It is high, but standard with these parameters (Hornstein et al., 2005).

We vary the simulations according to three main parameters. First, heterogeneity is introduced. We are agnostic about the shape of the production function and use a linear specification, $y(x) = 1 - d.x$, with d the slope of the productivity function. The best match has a productivity equal to 1, to compare with the benchmark model. Second, the elasticity η and the bargaining parameter β play a crucial role in the optimal tax formula. Whereas there are many estimates of η in the literature (Petrongolo and Pissarides, 2001), there is no uncontroversial empirical counterpart to the bargaining power β. For instance,

[10] A discussion of possible calibration for the benchmark search and matching model is provided by Hornstein et al. (2005).

Table 1: Parameters for the calibration

Parameter	**Target/Motive**	**Value**
Discount rate, r	Annual interest rate of 5%	0.012
Exogenous job destruction rate, s	10% of jobs break each quarter	0.1
Matching function $m(\theta)$	Multiplicative factor normalized to 1, elasticity from Shimer (2005)	$\theta^{0.25}$
Search function $s(e)$	Job-finding rate at 1.35 (Shimer, 2005), elasticity from Hornstein et al. (2011)	$s(e) = 2.59e^{0.5}$
Vacancy cost, k	Normalize $\theta = 1$	0.13
Home production, h	Unemployment payoff equal to 0.4 (Shimer, 2005)	0.67
Bargaining parameter, β	Hosios-Pissarides condition	0.75

Note: The time unit is a quarter. Productivity is normalized to 1.

Cahuc et al. (2006) find variation of this parameter according to worker's education. The elasticity takes the values 0.25 and 0.5, higher estimates are less plausible according to the survey of Petrongolo and Pissarides (2001). We let the bargaining power takes value 0.25, 0.5 and 0.75. We let the slope of the production function to go from 0 to 1. When $d = 1$, the best match is twice more productive than the worst feasible match, $y(1/2) = 0.5$. For each set of parameters, we compute *laissez-faire* and efficient outcomes. Results are given in table 2. The benchmark model without mismatch is the first line of the table ($\beta = 0.75$ and $d = 0$). Remember that the social planner maximizes the value of unemployment, ψ. Comparing the welfare is equivalent to comparing ψ in the case with and without optimal taxes. Following Hornstein et al. (2011), the mean-min wage ratio Mm is computed. The Mean-min ratio is $\mathbb{E}_{x|\mathbf{e}} w_x/\psi$ in our case as ψ is the reservation wage.

Switching from the *laissez-faire* decentralized equilibrium to the efficient allocation can significantly increase welfare in some circumstances. The welfare loss strongly depends on the bargaining parameter. It is the highest when workers have low bargaining power and when the elasticity of the matching function is low. It is indeed the worst case scenario. The surplus sharing is the least efficient given $1 - \beta \neq \eta$, and the wage curve is flat. The second observation is that the introduction of mismatch seems to contribute to the welfare loss as equally as the failure of the Hosios-Pissarides condition ($\eta \neq 1 - \beta$). The

Table 2: Simulated outcomes for different matching elasticities, bargaining parameters and slopes of the production function

Matching elasticity $\eta = 0.25$

Bargaining β	Slope prod. d	Welf. loss $\frac{\psi^o - \psi^e}{\psi^o}$	Unemployment u^e	u^o	Mm ratio Mm^e	Mm^o
	0	0	7.2%	7.2%	1.05	1.05
0.75*	0.5	0.1%	11.9%	12.70%	1.07	1.07
	1	0.1%	14.9%	15.9%	1.08	1.09
	0	0.8%	5.7%	7.2%	1.03	1.05
0.5	0.5	1.2%	9.3%	12.7%	1.05	1.07
	1	1.4%	11.9%	15.9%	1.06	1.09
	0	3.9%	4.7%	7.2%	1.03	1.05
0.25	0.5	5.3%	7.4%	12.7%	1.03	1.07
	1	6%	9.8%	15.9%	1.04	1.09

Matching elasticity $\eta = 0.5$

Bargaining β	Slope prod. d	Welf. loss $\frac{\psi^o - \psi^e}{\psi^o}$	Unemployment u^e	u^o	Mm ratio Mm^e	Mm^o
	0	0.8%	7.2%	4.2%	1.05	1.03
0.75	0.5	1.6%	10.8%	7.1%	1.06	1.04
	1	3.5%	12.8%	14.2%	1.07	1.04
	0	0	4.2%	4.2%	1.03	1.03
0.5*	0.5	0.4%	6.5%	7.1%	1.04	1.04
	1	2.2%	7.7%	14.2%	1.04	1.04
	0	0.8%	2.5%	4.2%	1.02	1.03
0.25	0.5	1.5%	3.9%	7.1%	1.02	1.04
	1	2.9%	5.5%	14.2%	1.02	1.04

Note. For each tuple (η, β, d), we simulate values of unemployment ψ, unemployment rates u and Mm ratios at the *laissez-faire* equilibrium and at the optimal level, respectively with the superscripts e and o. The * notation indicates the case when the Hosios-Pissarides condition is satisfied.

welfare loss when the production function is constant is always below 1% except in the case $\eta = 0.25$ and $\beta = 0.25$ when it reaches 3.9%.

The conclusions of Hornstein et al. (2011) that search frictions hardly explain residual wage dispersion hold, despite the presence of mismatch. The Mm ratio is always below 1.1, whereas the empirical ratio suggests it should be between 1.7 and 2. The reason is that the value of unemployment ψ is high, which induces workers to be selective when applying to jobs. The slope of the production function turns out to have very little effect on the Mm ratio.

Intuitively, we could expect the unemployment rate to be higher at the optimum than at the *laissez-faire* equilibrium because workers might reduce their job-finding rate to be more selective. This is not always true. By increasing her search efforts on each segment and searching even more for high-productive jobs, it is possible for a worker to be more selective and to have a higher job-finding rate.

4 Other policy instruments

The combination of a linear tax on production and a linear wage tax is especially relevant since a policy maker only requires to know the elasticity of the matching function and the bargaining power. This section studies the efficiency of other public policies: unemployment benefits, non-linear income taxation, a subsidy to job creation and minimum wage.

4.1 Unemployment benefits

Consider the same theoretical framework with unemployment benefits b. Taxes on employment are now used to finance these unemployment benefits,

$$\mathbb{E}_{x|\mathbf{e}} T_x (1 - u) = bu.$$

The introduction of unemployment benefits modifies equation (4). The unemployed workers receive $h + b$ instead of h. The value of unemployment ψ now differs between the efficient allocation and the decentralized equilibrium. With unemployment benefits, ψ is increased by $\frac{r+\delta}{r+\delta+m(\theta)\int_0^{1/2} s(e_x)2dx}b < b$. It is less than b because unemployment benefits reduce the gains from matching $W_x - U$. Keeping ψ as defined in equation (1), the wage is characterized by the equivalent of Lemma 2:

$$\mathbb{E}_{x|\mathbf{e}} w_x = \beta^w \mathbb{E}_{x|\mathbf{e}}(y_x - T_x) + (1-\beta^w)\left(\psi + \frac{r+\delta}{r+\delta+m(\theta)\int_0^{1/2} s(e_x)2dx} b\right), \quad (19)$$

$$w_x - \mathbb{E}_{x|\mathbf{e}} w_x = \beta^s \left(y_x - \mathbb{E}_{x|\mathbf{e}} y_x\right). \quad (20)$$

Unemployment benefits modify the surplus sharing but do not affect relative wages, $w_x - w_{x'}$. A decentralized equilibrium is efficient when the two conditions are met:

$$(1-\beta^w-\eta)\frac{\mathbb{E}_{x|\mathbf{e}} y_x - \psi}{r+\delta} = (1-\beta^w)\frac{b}{r+\delta+m(\theta)\int_0^{1/2} s(e_x)2dx} + (1-\beta^w)\frac{\mathbb{E}_{x|\mathbf{e}} T_x}{r+\delta}, \quad (21)$$

$$(1-\beta^s)\left(y_x - \mathbb{E}_{x|\mathbf{e}} y_x\right) = -\frac{r+\delta}{r+\delta+m(\theta)\int_0^{1/2} s(e_x)2dx} b - \mathbb{E}_{x|\mathbf{e}} T_x, \qquad \forall x \in (0, 1/2). \quad (22)$$

The two conditions generalize Proposition 2 with unemployment benefits. Without unemployment benefits, the right-hand sides are equal to zero. Because the right-hand side of the second condition does not depend on x, it is necessary that $\beta^s = 1$ and so $b = 0$. Consequently, unemployment benefits are useless to decentralize the first-best allocation. Unemployment benefits do not affect the slope of the wage curve, so that the optimal wage profile is unreachable. The social planner would ideally want benefits that are conditional on match-specific search activities, b_x. This is similar to the search tax that Shimer and Smith (2001) use to illustrate the inefficiency of their equilibrium.

In absence of a production tax, unemployment benefits, however, may be useful to decentralize a second-best allocation that we do not define here. First, unemployment benefits could play a role in the surplus sharing regarding the first condition. An increase in unemployment benefits raises the worker's share of the surplus. Second, the effect on search activities is ambiguous. Higher unemployment benefits make workers more selective, but they also reduce the search efforts through the moral hazard channel. The social planner would like search efforts to increase for high-type jobs and to decrease for low-type jobs.

4.2 Non-linear income taxation

The production tax τ^y might only be needed because the income taxation is constrained to be linear. Here, we relax this hypothesis and consider an income tax $T^w(w_x)$ instead of $\tau^w w_x$. We show that the optimal income taxation is linear and the production tax is necessary.

Without loss of generality, the tax τ^f can be normalized to 0 as lump-sum transfers are possible through T^w. The new wage bargain involves the derivative of the tax function, $T^{w\prime}$,

$$w_x - \psi = \frac{\beta}{1 + (1-\beta)T^{w\prime}(w_x)} \left(y_x - \psi - \tau^y y_x - T^w(w_x)\right). \tag{23}$$

The problem of optimal taxation consists in finding T^w and τ^y such that:

$$w_x = y_x - (r+\delta)\frac{k}{q(\theta)} \tag{24}$$

$$\mathbb{E}_{x|\mathbf{e}}(\tau^y y_x + T^w(w_x)) = 0 \tag{25}$$

The first equation is the definition of the efficient wage profile in Proposition 1. The government's budget constraint is balanced when total tax receipts are nil; this is the second condition. Suppose T^w makes (24) true. One can substitute y_x in equation (23). The optimal wage tax must be a solution of the following differential equation:

$$T^{w\prime}(w) + \frac{\beta}{(1-\beta)(w-\psi)} T^w(w) = \beta \frac{\tau^y \psi + (1-\tau^y)(r+\delta)\frac{k}{q(\theta)}}{(1-\beta)(w-\psi)} - \frac{1-\beta+\beta\tau^y}{1-\beta}. \tag{26}$$

T^w should be of the form $T^w(w) = Z_0(w-\psi)^{-\beta/(1-\beta)} + Z_1(w-\psi) + Z_2$. It requires $Z_1 = -(1-\beta+\beta\tau^y)$, $Z_2 = \tau^y\psi + (1-\tau^y)(r+\delta)\frac{k}{q(\theta)}$ and Z_0 is a free parameter. As long as the taxation function is bounded, Z_0 is necessarily equal to 0.[11] The optimal taxation

[11] When $Z_0 \neq 0$, the absolute value of $T^w(\psi)$ tends to infinity and so do profits. The bargaining strategy

function is thus linear, and we can check that $\tau^w = Z_1$ and τ^y derives from condition (25).

4.3 Subsidizing job creation

Workers' search strategy impacts job creation through the composition externality. It might be relevant to subsidize job creation to compensate for the profits loss due to mismatch. Here, we show that subsidizing job creation only affects the first inefficiency and cannot help making job seekers more selective.

In addition to the baseline tax scheme, the fiscal authority provides a financial aid a for each vacancy when the job is filled. The expected vacancy cost is thus reduced by a, $\frac{k}{q(\theta)} - a$. The budget constraint equates tax revenues and expenditures, $\mathbb{E}_{x|\mathbf{e}} T_x (1-u) = aq(\theta)v$. The wage bargaining is not different from the characterization in Lemma 2, except that $\mathbb{E}_{x|\mathbf{e}} T_x$ may not be zero:

$$\mathbb{E}_{x|\mathbf{e}} w_x = \beta^w \mathbb{E}_{x|\mathbf{e}} (y_x - T_x) + (1-\beta^w)\psi, \tag{27}$$

$$w_x - \mathbb{E}_{x|\mathbf{e}} w_x = \beta^s \left(y_x - \mathbb{E}_{x|\mathbf{e}} y_x \right). \tag{28}$$

A decentralized equilibrium is efficient when the two conditions are met:

$$(1-\beta^w - \eta) \frac{\mathbb{E}_{x|\mathbf{e}} y_x - \psi}{r+\delta} = (1-\beta^w) \frac{\mathbb{E}_{x|\mathbf{e}} T_x}{r+\delta} - a, \tag{29}$$

$$(1-\beta^s) \left(y_x - \mathbb{E}_{x|\mathbf{e}} y_x \right) = -\mathbb{E}_{x|\mathbf{e}} T_x - (r+\delta)a, \qquad \forall x \in (0, 1/2). \tag{30}$$

The analysis is analogous to the case of unemployment benefits. The two conditions generalize Proposition 2 with a subsidy to job creation. Again, $\beta^s = 1$ is necessary as the financial aid does not depend on the type x. The subsidy can help decentralizing an efficient surplus sharing but won't provide incentives for workers to be more selective. The production tax is necessary.

of firms would be degenerated.

4.4 Minimum wage

In an equilibrium without taxes ($\tau^w = \tau^y = 0$), a minimum wage $\underline{w}$ is equivalent to banning jobs that are not productive enough, namely when productivity y_x is below $\underline{w}$. A minimum wage can thus be useful when workers search where they should exert no effort at all, meaning when $e_x > 0$ in the decentralized equilibrium but $e_x = 0$ in the efficient allocation. The wage bargaining, however, remains unchanged as well as the slope of the wage curve. Workers still exert insufficient search efforts for high-productive jobs. In the lens of the model, a minimum wage can only decentralize the first part of the optimal search strategy in equation (3). This result is sufficient in the special case where search is only an extensive margin (a binary decision for each market) as in the model of Decreuse (2008). Otherwise, the composition externality cannot be internalized.

5 Discussion

My model is a limit case when heterogeneity can be reduced to one dimension exhibiting symmetry properties, and risk-neutrality makes inequalities irrelevant to the social planner. In this section, I discuss how the findings of the paper change with less restrictive assumptions.

5.1 Risk aversion

Risk aversion introduces two new motives for the social planner. First, the randomness of the search process creates a wage risk. More dispersed wages make workers worse-off. Second, workers suffer from the income loss when unemployed, the unemployment risk. There is thus a need for both redistribution as for Golosov et al. (2013), and unemployment insurance as for Landais et al. (2010). The first-best allocation with risk aversion is characterized by the same equilibrium distribution $\mathcal{D}$ of a first-best allocation with risk-neutrality. The difference is that there is a unique wage profile, the one that equalizes consumption in any state.

The objective of redistribution goes against the objective of efficiency that requires a steep wage profile. To maximize efficiency, the social planner would like the wage profile with a slope equal to 1 with production. To maximize equity, it would like a constant wage profile. Consequently, there is no equivalent of Proposition 1 with risk-averse workers. In other words, there does not exist a wage profile that decentralizes the first-best equilibrium with risk aversion. If the social planner wants to maximize welfare with a control on the wage profile, it can only reach a second-best allocation.

Because of the failure of Proposition 1, the tax scheme with proportional tax rates cannot decentralize the first-best equilibrium. Even if Proposition 1 were true, risk aversion would make the Nash bargaining more complicated. In particular, Lehmann and Linden (2007) determine the optimal (second-best) nonlinear income taxes with risk-averse workers without heterogeneity.

5.2 Richer heterogeneity

In the paper, heterogeneity is critically reduced through the assumption of symmetric comparative advantages. Proposition 1 holds in the general case, but not Proposition 2. With richer heterogeneity, the social planner would give incentives for high-skilled workers to search more and for low-skilled workers to search less as Shimer and Smith (2001) suggest. A job application from a high-skilled worker instead of a low-skilled worker improves the expected productivity, which improves profits and job creation.

I do not solve the model here, but two features seem necessary to extend the tax scheme of the paper to unconstrained two-sided heterogeneity. First, it is important that anti-redistribution occurs within each industry j. In other words, there should be redistribution from low-type matches to high-type matches *within* each segment of the market. Consequently, each worker would search efficiently. In the paper, each segment is symmetric so this condition is met. Second, it might still be necessary to observe workers' type to subsidize high-skilled workers that generate positive composition externalities.

5.3 Directed search and posted wages

The results in this paper rely on the hypothesis of Nash-bargained wages. The alternative wage setting in many search and matching models is posted wages. Moen (1997) shows that this setting decentralizes the efficient allocation. This efficiency property holds for many extensions, as shown by Acemoglu and Shimer (1999) for instance.

It may hold here if firms are able to commit to a wage contingent to each type of workers. The posting strategy of a firm would be equivalent to a mapping from I to $\mathbb{R}^+$, defining a wage for any $i \in I$. The reason why posted wages may decentralize the first-best allocation in this framework is intuitive. By posting a wage for each type i, firms actually increase the number of segments of the labor market. Instead of having a segment per firm type j in the Nash-bargained framework, there is a segment for each pair (i, j) at equilibrium with posted wages. Firms thus manage to circumvent part of the search and information frictions. Wages for each segment (i, j) would satisfy the second property of Proposition 1, that ex-ante match surpluses are equal to zero.

The agents' ability to extend the number of segments may not be reasonable if I is a large set, as it supposes firms pay no cost of engineering a wage contract. It also supposes that i-type workers do not direct their search efforts to segments of the market dedicated to i'-type workers as emphasized by Acemoglu and Shimer (1999).

6 Conclusion

This paper examines the optimal fiscal policy in presence of inefficient job mismatch due to a composition externality. It sheds light on a harmful consequence of redistributive taxation, due to the presence of inefficient mismatch. The selectivity of job seekers matters for firms as it determines the quality of jobs. Job seekers, however, do not internalize this composition externality and make suboptimal search decisions. Redistribution thus amplifies this inefficiency. To emphasize the main mechanism, the model abstracts from a variety of considerations such as risk aversion or (asymmetric) heterogeneity in skills. The paper is not aimed at recommending such an anti-redistributive taxation on the labor market.

In practice, whether the composition externality is large or not depends on two key channels: i) how job creation responds to job productivity; ii) how the mismatch reduces job productivity. The optimal policy, however, does not depend on these parameters.

The composition externality shares similar mechanisms with holdup problems. In particular, the tax scheme suggested here decentralizes the social optimum in a search and matching model with homogeneous agents and workers' investments. This paper also provides a general recipe for holdup in firms' investments, in which the slope of the wage curve should be substituted by the slope of the profits curve in the analysis.

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A Lemma 1

Let $H(\theta, \mathbf{e}, \{n_x\}_{x\in(0,1/2)}, u, \{\lambda_x\}_{x\in(0,1/2)}, \mu)$ be the current-value Hamiltonian of the planner's program, with λ_x and μ the multipliers of the constraints on n_x and u. Hereafter, we denote it H for ease of reading. It is defined as

$$\begin{aligned}H = & \int_0^{1/2} y_x n_x 2dx + hu - kv - \left(\int_0^{1/2} e_x 2dx\right) u \\ & + \int_0^{1/2} \lambda_x \left[s(e_x)m(\theta)u - \delta n_x\right] 2dx \\ & + \mu \left[\delta\,(1-u) - \left(\int_0^{1/2} s(e_x)m(\theta)2dx\right) u\right].\end{aligned}$$

The two costate equations at steady state write $\frac{\partial H}{\partial n_x} = r\lambda_x$ and $\frac{\partial H}{\partial u} = r\mu$. The costate equation associated with the output dynamics determines the shadow price of a x-type job, λ_x, as

$$\lambda_x = \frac{y_x}{r+\delta}.$$

Instead of using λ_x and μ as values of employment and unemployment, we use equivalently production $y(x)$ and ψ, by defining $\psi \equiv (r+\delta)\mu$. Equation (1) derives from the second costate equation, after having substituted λ_x by its expression.

The optimal market tightness, θ, and search strategy, ξ, maximize the Hamitonian H. the Hamiltonian is linear with the state variables: $H = \int_0^{1/2} \frac{\partial H}{\partial n_x} n_x 2dx + \frac{\partial H}{\partial u} u$ where the integral term does not depend on θ and ξ. The planner's problem is thus equivalent to maximizing $\frac{\partial H}{\partial u}$ i.e. the value of an unemployed worker μ according to the second costate equation. The planner thus maximize ψ in Lemma 1.

B Uniqueness of an efficient allocation

The value of unemployment ψ is defined as a function of θ and $\mathbf{e}$ by

$$\begin{aligned}\psi = & \frac{r+\delta}{r+\delta+\int s(e_x)2dx.m(\theta)}\left(h - \int e_x 2dx\right) \\ & + \int_0^{1/2} \left\{\frac{s(e_x)m(\theta)}{r+\delta+\int s(e_{x'})2dx'.m(\theta)}\left[y_x - (r+\delta)\frac{k}{q(\theta)}\right]\right\} 2dx. \qquad (31)\end{aligned}$$

Optimal market tightness θ and optimal search strategy $\mathbf{e}$ maximizes ψ in equation

(1). The objective function to maximize rewrite

$$(\theta, \mathbf{e}) \mapsto \int_0^{1/2} \left\{ s(e_x)m(\theta)\left[\frac{y_x - \psi}{r+\delta} - \frac{k}{q(\theta)}\right] - e_x \right\} 2dx.$$

This function is concave because all the functions $(\theta, e_x) \mapsto s(e_x)m(\theta)\left[\frac{y_x-\psi}{r+\delta} - \frac{k}{q(\theta)}\right]$ are concave. Then, for any ψ, there exists a unique pair $(\theta, \mathbf{e})$ solution of the first-order conditions (2) and (3) as a global maximum. Finally, assume there are two efficient allocations $(\psi_1, \theta_1, \mathbf{e}_1)$ and $(\psi_2, \theta_2, \mathbf{e}_2)$. If $\psi_1 < \psi_2$, then the pair $(\theta_1, \mathbf{e}_1)$ cannot be a solution because the pair should also maximize (31) thanks to the envelop condition. Consequently, the first-order conditions are sufficient to obtain an efficient allocation.

C Lemma 2

The ex-post surplus of a x-type match is $\Omega(x) \equiv J(x) + W(x) - U$. The first-order condition makes apparent the worker's share of the surplus β^w as defined in Lemma 2,

$$W(x) - U = \beta^w \Omega(x). \tag{32}$$

First, equation (13) derives from the expectation, $\mathbb{E}_{x|\mathbf{e}} W_x - U = \beta^w \mathbb{E}_{x|\mathbf{e}} \Omega_x$. Second, equation (14) derives from the difference with the expectation, $W(x) - \bar{W}(\xi) = \beta^w(\Omega(x) - \bar{\Omega}(\xi))$. The slope of the wage curve, β^s, appears as a combination of β^w, τ^y and τ^w

D Lemma 3

We prove the second property of Lemma 3. Set θ and ψ as fixed, and denote $\mathbf{e}^{op}$ a solution of (3) and $\mathbf{e}^{eq}$ a solution of (16). Comparing (3) and (16) yields the equivalence:

$$y_x \leq \mathbb{E}_{x|\mathbf{e}^{eq}} y_x \Leftrightarrow e_x^{eq} \geq e_x^{op}.$$

s is increasing and one can prove that $s(e^{eq}_{x'})\left[y_{x'} - \mathbb{E}_{x|\mathbf{e}^{eq}} y_x\right] \leq s(e^{op}_{x'})\left[y_{x'} - \mathbb{E}_{x|\mathbf{e}^{eq}} y_x\right]$, no matter the sign of $y_{x'} - \mathbb{E}_{x|\mathbf{e}^{eq}} y_x$. After integration,

$$\int_0^{\frac{1}{2}} s(e^{eq}_{x'})\left[y_{x'} - \mathbb{E}_{x|\mathbf{e}^{eq}} y_x\right] 2dx' \leq \int_0^{\frac{1}{2}} s(e^{op}_{x'})\left[y_{x'} - \mathbb{E}_{x|\mathbf{e}^{eq}} y_x\right] 2dx'.$$

By definition of $\mathbb{E}_{x|\mathbf{e}^{eq}} y_x$, the left-hand side is equal to 0. It follows

$$\int_0^{\frac{1}{2}} s(e^{op}_{x'}) \mathbb{E}_{x|\mathbf{e}^{eq}} y_x 2dx' \leq \int_0^{\frac{1}{2}} s(e^{op}_{x'}) y_{x'} 2dx'$$

and, lastly,

$$\mathbb{E}_{x|\mathbf{e}^{eq}} y_x \leq \mathbb{E}_{x|\mathbf{e}^{op}} y_x.$$