Introduction to Structural Econometrics

Lesson 3: Monte-Carlo Methods

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Schedule

- October 3rd, room 23, 10-12am
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- October 10th, room 23, 10-12am
  Dynamic Programming and Recursive Models
- October 24th, room 23, 10-12am
  Monte-Carlo Methods
- November 7th, room 23, 10-12am
  Simulation-based Estimation
- November 14th, room 15, 10-12am
  State-Space Representation and Latent Variables
- November 21st, room 16, 10-12am
  Numerical tools

Plan

Improve numerical performance

1. Monte-Carlo methods enables econmetricians to estimate models that are numerically demanding
2. A typical example is a *n*-dimensional integral
3. for low values of *n*, the numerical tool is the quadrature
    1. discretize the space of integration
    2. approximate the integral by a sum

Theory (I)

- Consider a c.d.f. $\mathcal{F}$, possibly multivariate
- The goal is to estimate

$$\mathbb{E}[g(X)] = \int g(x) d\mathcal{F}(x) = \int g(x) f(x) dx$$

- We just need to know how to
  - evaluate $g(.)$
  - compute an average
  - sample random variables from $\mathcal{F}$
- We will not evaluate $f(.)$ or $\mathcal{F}(.)$ at any time*

Theory (II)

- Take N independent variables X_1, X_2, ..., X_N drawn from $\mathcal{F}$,
- then the average $\tilde{g}_N = \frac{1}{N}(g(X_1) + g(X_2) + ... + g(X_N))$ converges to $\mathbb{E}[g(X)]$
- True, according to Law of Large Numbers
- We can know the error with the Central Limit Theorem

Importance Sampling

- Suppose $g(x) = 0$ on a large part of support of $\mathcal{F}$
- When we will draw a X_i, there will be a high probability that the observation will not be "useful"
- The idea of importance sampling is to estimate the integral using another distribution to "weight" observations

Importance Sampling (II)

- For instance, suppose we know a distribution $(\mathcal{H}, h)$ such that $g(x) = 0$ occurs "less often" on the support of the new distribution
- We can write

$$\mathbb{E}[g(X)] = \int g(x) \frac{f(x)}{h(x)} d\mathcal{H}(x)$$

- Contrary to before,
  1. sample variables from $\mathcal{H}$ instead of $\mathcal{F}$
  2. evaluate $g(X_i)f(X_i)/h(X_i)$ instead of $g(X_i)$

Markov Chain Monte-Carlo (MCMC)

- When you draw a sequence $X_1, ..., X_N$, you can be unlucky...
- The idea of MCMC is to "correct" the sequence as you draw a new variable
- Use the information on $X_1, .., X_i$ to draw X_{i+1}, etc
- You can improve the convergence property of the sequence
- Two famous methods: Metropolis-Hastings and Gibbs sampling

Plan

Estimate Pi

- A classical example of Monte-Carlo methods is the estimation of π as the solution of a bidimensional integral
- We know that π is the area of a disk of radius 1. Analytically, it writes:

$$\pi = \int \int \mathbf{1}(x^2 + y^2 < 1) dxdy. \quad (1)$$

Geweke-Hajivassiliou-Keane simulator

- Consider the problem of panel data with binary dependent variable

$$y_{it} = \beta x_{it} + \epsilon_{it} \quad (2)$$

$$d_{it} = \mathbf{1}(y_{it} > 0) \quad (3)$$

- the difficulty here is when $\epsilon_i = (\epsilon_{i1}, .., \epsilon_{iT}) \sim \mathcal{N}(0_T, \Sigma)$.

GHK (II)

- As we assume uncorrelated errors across individuals, we can focus on the likelihood of an individual.
- The probability to observe the sequence D_i given X_i is

$$\mathbb{P}(D_i|X_i) = \int_{\epsilon_{i1}} \dots \int_{\epsilon_{iT}} h(\epsilon_{i1}, .., \epsilon_{iT}) d\mathcal{F}_{\Sigma}(\epsilon_{i1}, .., \epsilon_{iT}) \tag{4}$$

- h is a 0-1 function

GHK (III)

- $h(\epsilon_{i1}, .., \epsilon_{iT}) = 1$ if and only if

$$\begin{aligned} a_{i1} < \epsilon_{i1} < b_{i1}, \\ ..., \\ a_{iT} < \epsilon_{iT} < b_{iT}. \end{aligned}$$

with possibly $a_{it} = -\infty$ or $b_{it} = \infty$.

Can we do better than the "crude" Monte-Carlo?

Yes, application